Analysis of variance (ANOVA)

One Way ANOVA

Analysis of variance (ANOVA) is a hypothesis-testing procedure that is used to evaluate

mean differences between two or more treatments (or populations).

1. There really are no differences between the populations (or treatments). The observed differences between the sample means are caused by random, unsystematic factors (sampling error) that differentiate one sample from another.

2. The populations (or treatments) really do have different means, and these population mean differences are responsible for causing systematic differences between the sample means.

DEFINITION:In ANOVA, the variable (independent or quasi-independent) that designates the groups being compared is called a **factor**.

DEFINITION:The individual conditions or values that make up a factor are called the **levels** of the factor

DEFINITION:The **testwise alpha level** is the risk of a Type I error, or alpha level, for an individual hypothesis test.

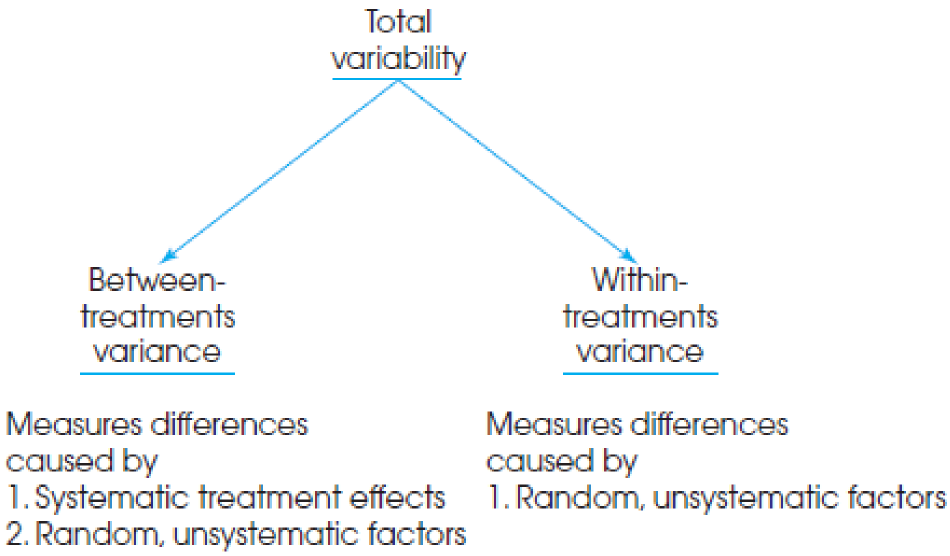
DEFINITION:When an experiment involves several different hypothesis tests, the **experimentwise alpha level** is the total probability of a Type I error that is accumulated from all of the individual tests in the experiment. Typically, the experimentwise alpha level is substantially greater than the value of alpha used for any one of the individual tests.

DEFINITION(**BETWEEN-TREATMENTS VARIANCE**)Remember that calculating variance is simply a method for measuring how big the differences are for a set of numbers. When you see the term *variance,* you can automatically translate it into the term *differences*. Thus, the *between-treatments variance* simply measures how much difference exists between the treatment conditions. There are two possible explanations for these between-treatment differences:

1. The differences between treatments are not caused by any treatment effect but are simply the naturally occurring, random, and unsystematic differences that exist between one sample and another. That is, the differences are the result of sampling error.

2. The differences between treatments have been caused by the *treatment effects*

DEFINITION(**WITHIN-TREATMENTS VARIANCE**)Inside each treatment condition, we have a set of individuals who all receive exactly the same treatment; that is, the researcher does not do anything that would cause these individuals to have different scores.



$$F = \frac{\text{systematic treatment effects} + \text{random, unsystematic differences}}{\text{random, unsystematic differences}}$$

DEFINITION: For ANOVA, the denominator of the *F*-ratio is called the **error term.** The error term provides a measure of the variance caused by random, unsystematic differences. When the treatment effect is zero (H_0 is true), the error term measures the same sources of variance as the numerator of the *F*-ratio, so the value of the *F*-ratio is expected to be nearly equal to 1.00.

Exercises

1. Explain the difference between the testwise alpha level and the experimentwise alpha level.

2. The term "analysis" means separating or breaking a whole into parts. What is the basic analysis that takes place in analysis of variance?

3. If there is no systematic treatment effect, then what value is expected, on average, for the *F*-ratio in an ANOVA?

4. What is the implication when an ANOVA produces a very large value for the *F*-ratio?

Answers

1. When a single research study involves several hypothesis tests, the testwise alpha level is the value selected for each individual test and the experimentwise alpha level is the total risk of a Type I error that is accumulated for all of the separate tests.

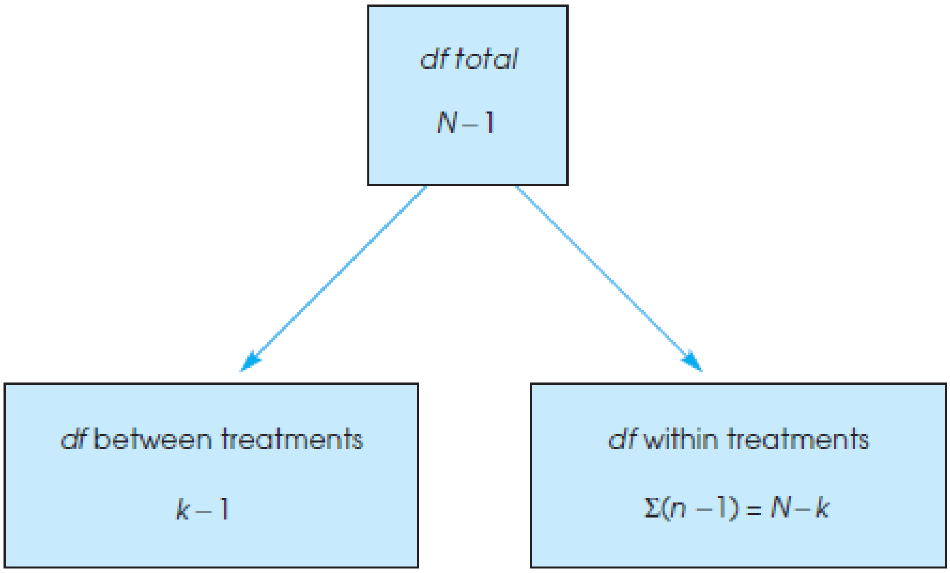
2. In ANOVA, the total variability for a set of scores is separated into two components: between-treatments variability and within-treatments variability.

3. When H_0 is true, the expected value for the *F*-ratio is 1.00 because the top and bottom of the ratio are both measuring the same variance.

4. A large *F*-ratio indicates the existence of a treatment effect because the differences between treatments (numerator) are much bigger than the differences that would be expected if there were no effect (denominator).

ANOVA Notation and Formulas

The final goal for the ANOVA is an *F*-ratio	$F = \frac{\text{Variance between treatments}}{\text{Variance within treatments}}$	
Each variance in the *F*-ratio is computed as *SS*/*df*	Variance between treatments $= \frac{SS \text{ between}}{df \text{ between}}$	Variance within treatments $= \frac{SS \text{ within}}{df \text{ within}}$
To obtain each of the *SS* and *df* values, the total variability is analyzed into the two components	*SS* total *SS* between *SS* within	*df* total *df* between *df* within



$$SS_{\text{total}} = \Sigma X^2 - \frac{G^2}{N} \qquad df_{\text{total}} = N - 1$$

$$SS_{\text{between}} = \Sigma \frac{T^2}{n} - \frac{G^2}{N} \qquad df_{\text{between}} = k - 1$$

$$SS_{\text{within}} = \Sigma SS_{\text{inside each treatment}} \qquad df_{\text{within}} = N - k$$

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}} \qquad \text{where each } MS = \frac{SS}{df}$$

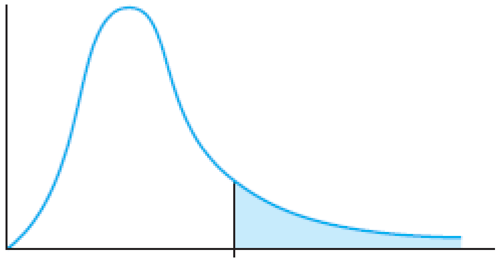
The Distribution of *F*-Ratios

In ANOVA, the *F*-ratio is constructed so that the numerator and denominator of the ratio are measuring exactly the same variance when the null hypothesis is true. In this situation, we expect the value of *F* to be around 1.00. If we obtain an *F*-ratio that is much greater than 1.00, then it is evidence that a treatment effect exists and the null hypothesis is false. The problem now is to define precisely which values are "around 1.00" and which are "much greater than 1.00." To answer this question, we need to look at all of the possible *F* values when H_0 is true—that is, the *distribution of* F*-ratios.*

Before we examine this distribution in detail, you should note two obvious characteristics:

1. Because *F*-ratios are computed from two variances (the numerator and denominator of the ratio), *F* values always are positive numbers. Remember that variance is always positive.

2. When H_0 is true, the numerator and denominator of the *F*-ratio are measuring the same variance. In this case, the two sample variances should be about the same size, so the ratio should be near 1. In other words, the distribution of *F*-ratios should pile up around 1.00.



With these two factors in mind, we can sketch the distribution of *F*-ratios. The distribution is cut off at zero (all positive values), piles up around 1.00, and then tapers off to the right . The exact shape of the *F* distribution depends on the degrees of freedom for the two variances in the *F*-ratio. You should recall that the precision of a sample variance depends on the number of scores or the degrees of freedom. In general, the variance for a large sample (large *df*) provides a more accurate estimate of the population variance. Because the precision of the *MS* values depends on *df*, the shape of the *F* distribution also depends on the *df* values for the numerator and denominator of the *F*-ratio. With very large *df* values, nearly all of the *F*-ratios are clustered very near to 1.00. With the smaller *df* values, the *F* distribution is more spread out.

EXAMPLE 1 :

A researcher is interested in the amount of homework required by different academic majors. Students are recruited from Biology, English, and Psychology to participate in the study. The researcher randomly selects one course that each student is currently taking and asks the student to record the amount of out-of-class work required each week for the course. The researcher used all of the volunteer participants, which resulted in unequal sample sizes. The data are summarized in Table

Biology	English	Psychology	
$n = 4$	$n = 10$	$n = 6$	$N = 20$
$M = 9$	$M = 13$	$M = 14$	$G = 250$
$T = 36$	$T = 130$	$T = 84$	$\Sigma X^2 = 3377$
$SS = 37$	$SS = 90$	$SS = 60$	

STEP 1: State the hypotheses, and select the alpha level.

$$H_0: \mu_1 = \mu_2 = \mu_3$$

H_1: At least one population is different.

$$\alpha = .05$$

STEP 2: Locate the critical region.

To find the critical region, we first must determine the *df* values for the *F*-ratio:

$$df_{total} = N - 1 = 20 - 1 = 19$$

$$df_{between} = k - 1 = 3 - 1 = 2$$

$$df_{within} = N - k = 20 - 3 = 17$$

The *F*-ratio for these data has $df = 2, 17$. With $\alpha = .05$, the critical value for the *F*-ratio is 3.59.

STEP 3: Compute the *F*-ratio.

First, compute the three *SS* values. As usual, SS_{total} is the *SS* for the total set of $N = 20$ scores, and SS_{within} combines the *SS* values from inside each of the treatment conditions.

$$SS_{total} = \Sigma X^2 - \frac{G^2}{N} = 3377 - 3125 = 252$$

$$SS_{within} = \Sigma SS_{\text{inside each treatment}} = 37 + 90 + 60 = 187$$

$SS_{between}$ can be found by subtraction (Equation 12.5).

$$SS_{between} = SS_{total} - SS_{within} = 252 - 187 = 65$$

Or, $SS_{between}$ can be calculated using the computation formula (see Equation 12.7). If you use the computational formula, be careful to match each treatment total (*T*) with the appropriate sample size (*n*) as follows:

$$SS_{between} = \Sigma\frac{T^2}{n} - \frac{G^2}{N}$$

$$= \frac{36^2}{4} + \frac{130^2}{10} + \frac{84^2}{6} - \frac{250^2}{20}$$

$$= 324 + 1690 + 1176 - 3125$$

$$= 65$$

Finally, compute the *MS* values and the *F*-ratio:

$$MS_{\text{between}} = \frac{SS}{df} = \frac{65}{2} = 32.5$$

$$MS_{\text{within}} = \frac{SS}{df} = \frac{187}{17} = 11$$

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}} = \frac{32.5}{11} = 2.95$$

STEP 4: Make a decision.

Because the obtained *F*-ratio is not in the critical region, we fail to reject the null hypothesis and conclude that there are no significant differences among the three populations of students in terms of the average amount of homework each week.

EXAMPLE 2 :

A human-factors psychologist studied three computer keyboard designs. Three samples of individuals were given material to type on a particular keyboard, and the number of errors committed by each participant was recorded. Are these following data sufficient to conclude that there are significant differences in typing performance among the three keyboard designs?

Keyboard A	Keyboard B	Keyboard C	
0	6	6	$N = 15$
4	8	5	$G = 60$
0	5	9	$\Sigma X^2 = 356$
1	4	4	
0	2	6	
$T = 5$	$T = 25$	$T = 30$	
$SS = 12$	$SS = 20$	$SS = 14$	

STEP 1 State the hypotheses, and specify the alpha level. The null hypothesis states that there are no differences among the keyboards in terms of number of errors committed. In symbols, we would state

$$H_0: \mu_1 = \mu_2 = \mu_3 \quad \text{(Type of keyboard used has no effect.)}$$

As noted previously in this chapter, there are a number of possible statements for the alternative hypothesis. Here we state the general alternative hypothesis:

H_1: At least one of the treatment means is different.

We set alpha at $\alpha = .05$.

STEP 2 Locate the critical region. To locate the critical region, we must obtain the values for df_{between} and df_{within}.

$$df_{\text{between}} = k - 1 = 3 - 1 = 2$$
$$df_{\text{within}} = N - k = 15 - 3 = 12$$

The F-ratio for this problem has $df = 2, 12$, and the critical F value for $\alpha = .05$ is $F = 3.88$.

STEP 3 Perform the analysis. The analysis involves the following steps:

1. Perform the analysis of *SS*.
2. Perform the analysis of *df*.
3. Calculate mean squares.
4. Calculate the F-ratio.

Perform the analysis of SS. We compute SS_{total} followed by its two components.

$$SS_{\text{total}} = \Sigma X^2 - \frac{G^2}{N} = 356 - \frac{60^2}{15} = 356 - \frac{3600}{15}$$
$$= 356 - 240 = 116$$

$$SS_{within} = \Sigma SS_{inside} \text{ each treatment}$$
$$= 12 + 20 + 14$$
$$= 46$$

By subtraction, $SS_{between} = SS_{total} - SS_{within} = 116 - 46 = 70$

Analyze degrees of freedom. We compute df_{total}. Its components, $df_{between}$ and df_{within}, were previously calculated (see step 2).

$$df_{total} = N - 1 = 15 - 1 = 14$$
$$df_{between} = 2$$
$$df_{within} = 12$$

Calculate the MS values. We determine the values for $MS_{between}$ and MS_{within}.

$$MS_{between} = \frac{SS_{between}}{df_{between}} = \frac{70}{2} = 35$$
$$MS_{within} = \frac{SS_{within}}{df_{within}} = \frac{46}{12} = 3.83$$

Compute the F-ratio. Finally, we can compute *F*.

$$F = \frac{MS_{between}}{MS_{within}} = \frac{35}{3.83} = 9.14$$

STEP 4 Make a decision about H_0, and state a conclusion. The obtained *F* of 9.14 exceeds the critical value of 3.88. Therefore, we can reject the null hypothesis. The type of keyboard used has a significant effect on the number of errors committed, $F(2, 12) = 9.14, p < .05$. The following table summarizes the results of the analysis:

Source	SS	df	MS	
Between treatments	70	2	35	$F = 9.14$
Within treatments	46	12	3.83	
Total	116	14		

COMPUTING EFFECT SIZE FOR ANOVA

We compute eta squared (η^2), the percentage of variance explained, for the data that were analyzed in Demonstration 12.1. The data produced a between-treatments *SS* of 70 and a total *SS* of 116. Thus,

$$\eta^2 = \frac{SS_{between}}{SS_{total}} = \frac{70}{116} = 0.60 \text{ (or 60\%)}$$

Post hoc tests

Post hoc tests (or **posttests**) are additional hypothesis tests that are done after an ANOVA to determine exactly which mean differences are significant and which are not. As the name implies, post hoc tests are done after an ANOVA. More specifically, these tests are done after ANOVA when

1. You reject H_0 and

2. There are three or more treatments ($k >= 3$).

- **TUKEY’S HONESTLY SIGNIFICANT DIFFERENCE (HSD) TEST**

The first post hoc test we consider is *Tukey’s HSD test.* We selected Tukey’s HSD test because it is a commonly used test in psychological research. Tukey’s test allows you to compute a single value that determines the minimum difference between treatment means that is necessary for significance. This value, called the *honestly significant difference,* or HSD, is then used to compare any two treatment conditions. If the mean difference exceeds Tukey’s HSD, then you conclude that there is a significant difference between the treatments. Otherwise, you cannot conclude that the treatments are significantly different.

- Tukey's procedure is only applicable for pairwise comparisons.
- It assumes independence of the observations being tested, as well as equal variation across observations (homoscedasticity or homogeinity of variance) .

- **THE SCHEFFÉ TEST**

Because it uses an extremely cautious method for reducing the risk of a Type I error, the *Scheffé test* has the distinction of being one of the safest of all possible post hoc tests (smallest risk of a Type I error). The Scheffé test uses an *F*-ratio to evaluate the significance of the difference between any two treatment conditions. The numerator of the *F*-ratio is an *MS*between that is calculated using *only the two treatments you want to compare*. The denominator is the same *MS*within that was used for the overall ANOVA. The “safety factor” for the Scheffé test comes from the following two considerations:

1. Although you are comparing only two treatments, the Scheffé test uses the value of *k* from the original experiment to compute *df* between treatments. Thus, *df* for the numerator of the *F*-ratio is k-1.

2. The critical value for the Scheffé *F*-ratio is the same as was used to evaluate the

F-ratio from the overall ANOVA. Thus, Scheffé requires that every posttest satisfy the same criterion that was used for the complete ANOVA.

The Relationship Between ANOVA and *t* Tests

This relationship can be explained by first looking at the structure of the formulas for *F* and *t*. The *t* statistic compares *distances:* the distance between two sample means (numerator) and the distance computed for the standard error (denominator). The *F*-ratio, on the other hand, compares *variances.* You should recall that variance is a measure of squared distance. Hence, the relationship: $F = t^2$.

Assumptions for the Independent-Measures ANOVA

The independent-measures ANOVA requires the same three assumptions that were

necessary for the independent-measures *t* hypothesis test:

1. The observations within each sample must be independent .

2. The populations from which the samples are selected must be normal.

3. The populations from which the samples are selected must have equal variances (homogeneity of variance).

Ordinarily, researchers are not overly concerned with the assumption of normality, especially when large samples are used, unless there are strong reasons to suspect that the assumption has not been satisfied. The assumption of homogeneity of variance is an important one. If a researcher suspects that it has been violated, it can be tested by Hartley's *F*-max test and Levene's test for homogeneity of variance.

EXERCISES

1.Explain why the *F*-ratio is expected to be near 1.00 when the null hypothesis is true.

2. Several factors influence the size of the *F*-ratio. For each of the following, indicate whether it influences the numerator or the denominator of the *F*-ratio, and indicate whether the size of the *F*-ratio would increase or decrease. In each case, assume that all other factors are held constant.

a. An increase in the differences between the sample means.

b. An increase in the size of the sample variances.

3. Why should you use ANOVA instead of several *t* tests to evaluate mean differences when an experiment consists of three or more treatment conditions?

4. Posttests are done after an ANOVA.

a. What is the purpose of posttests?

b. Explain why you do not need posttests if the analysis is comparing only two treatments.

c. Explain why you do not need posttests if the decision from the ANOVA is to fail to reject the null hypothesis.

5.A researcher reports an *F*-ratio with $df = 2, 27$ from an independent-measures research study.

a. How many treatment conditions were compared in the study?

b. What was the total number of participants in the study?

6. A research report from an independent-measures study states that there are significant differences between treatments, $F(3, 48) = 2.95$, $p < .05$.

a. How many treatment conditions were compared in the study?

b. What was the total number of participants in the study?

7. The following summary table presents the results from an ANOVA comparing three treatment conditions with $n = 8$ participants in each condition. Complete all missing values.

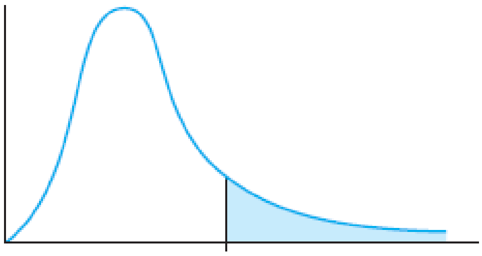
Source	*SS*	*df*	*MS*	
Between treatments	_____	_____	15	$F =$ _____
Within treatments	_____	_____	_____	
Total	93	____		

8. A pharmaceutical company has developed a drug that is expected to reduce hunger. To test the drug, two samples of rats are selected with $n = 20$ in each sample. The rats in the first sample receive the drug every day and those in the second sample are given a placebo. The dependent variable is the amount of food eaten by each rat over a 1-month period. An ANOVA is used to evaluate the difference between the two sample means and the results are reported in the following summary table. Fill in all missing values in the table.

Source	*SS*	*df*	*MS*	
Between treatments	____	____	20	$F = 4.00$
Within treatments	____	____	____	
Total	____	____		

TABLE B.4 THE *F* DISTRIBUTION*

*Table entries in lightface type are critical values for the .05 level of significance. Boldface type values are for the .01 level of significance.



Critical
F

Degrees of Freedom: Denominator	Degrees of Freedom: Numerator														
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*14*	*16*	*20*
1	161	200	216	225	230	234	237	239	241	242	243	244	245	246	248
	4052	**4999**	**5403**	**5625**	**5764**	**5859**	**5928**	**5981**	**6022**	**6056**	**6082**	**6106**	**6142**	**6169**	**6208**
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39	19.40	19.41	19.42	19.43	19.44
	98.49	**99.00**	**99.17**	**99.25**	**99.30**	**99.33**	**99.34**	**99.36**	**99.38**	**99.40**	**99.41**	**99.42**	**99.43**	**99.44**	**99.45**
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78	8.76	8.74	8.71	8.69	8.66
	34.12	**30.92**	**29.46**	**28.71**	**28.24**	**27.91**	**27.67**	**27.49**	**27.34**	**27.23**	**27.13**	**27.05**	**26.92**	**26.83**	**26.69**
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.93	5.91	5.87	5.84	5.80
	21.20	**18.00**	**16.69**	**15.98**	**15.52**	**15.21**	**14.98**	**14.80**	**14.66**	**14.54**	**14.45**	**14.37**	**14.24**	**14.15**	**14.02**
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74	4.70	4.68	4.64	4.60	4.56
	16.26	**13.27**	**12.06**	**11.39**	**10.97**	**10.67**	**10.45**	**10.27**	**10.15**	**10.05**	**9.96**	**9.89**	**9.77**	**9.68**	**9.55**
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.96	3.92	3.87
	13.74	**10.92**	**9.78**	**9.15**	**8.75**	**8.47**	**8.26**	**8.10**	**7.98**	**7.87**	**7.79**	**7.72**	**7.60**	**7.52**	**7.39**
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63	3.60	3.57	3.52	3.49	3.44
	12.25	**9.55**	**8.45**	**7.85**	**7.46**	**7.19**	**7.00**	**6.84**	**6.71**	**6.62**	**6.54**	**6.47**	**6.35**	**6.27**	**6.15**
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34	3.31	3.28	3.23	3.20	3.15
	11.26	**8.65**	**7.59**	**7.01**	**6.63**	**6.37**	**6.19**	**6.03**	**5.91**	**5.82**	**5.74**	**5.67**	**5.56**	**5.48**	**5.36**
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13	3.10	3.07	3.02	2.98	2.93
	10.56	**8.02**	**6.99**	**6.42**	**6.06**	**5.80**	**5.62**	**5.47**	**5.35**	**5.26**	**5.18**	**5.11**	**5.00**	**4.92**	**4.80**
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97	2.94	2.91	2.86	2.82	2.77
	10.04	**7.56**	**6.55**	**5.99**	**5.64**	**5.39**	**5.21**	**5.06**	**4.95**	**4.85**	**4.78**	**4.71**	**4.60**	**4.52**	**4.41**
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86	2.82	2.79	2.74	2.70	2.65
	9.65	**7.20**	**6.22**	**5.67**	**5.32**	**5.07**	**4.88**	**4.74**	**4.63**	**4.54**	**4.46**	**4.40**	**4.29**	**4.21**	**4.10**
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76	2.72	2.69	2.64	2.60	2.54
	9.33	**6.93**	**5.95**	**5.41**	**5.06**	**4.82**	**4.65**	**4.50**	**4.39**	**4.30**	**4.22**	**4.16**	**4.05**	**3.98**	**3.86**
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67	2.63	2.60	2.55	2.51	2.46
	9.07	**6.70**	**5.74**	**5.20**	**4.86**	**4.62**	**4.44**	**4.30**	**4.19**	**4.10**	**4.02**	**3.96**	**3.85**	**3.78**	**3.67**
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70	2.65	2.60	2.56	2.53	2.48	2.44	2.39
	8.86	**6.51**	**5.56**	**5.03**	**4.69**	**4.46**	**4.28**	**4.14**	**4.03**	**3.94**	**3.86**	**3.80**	**3.70**	**3.62**	**3.51**
15	4.54	3.68	3.29	3.06	2.90	2.79	2.70	2.64	2.59	2.55	2.51	2.48	2.43	2.39	2.33
	8.68	**6.36**	**5.42**	**4.89**	**4.56**	**4.32**	**4.14**	**4.00**	**3.89**	**3.80**	**3.73**	**3.67**	**3.56**	**3.48**	**3.36**
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.45	2.42	2.37	2.33	2.28
	8.53	**6.23**	**5.29**	**4.77**	**4.44**	**4.20**	**4.03**	**3.89**	**3.78**	**3.69**	**3.61**	**3.55**	**3.45**	**3.37**	**3.25**

TABLE B.4 (continued)

Degrees of Freedom: Denominator	Degrees of Freedom: Numerator 1	2	3	4	5	6	7	8	9	10	11	12	14	16	20
17	4.45	3.59	3.20	2.96	2.81	2.70	2.62	2.55	2.50	2.45	2.41	2.38	2.33	2.29	2.23
	8.40	**6.11**	**5.18**	**4.67**	**4.34**	**4.10**	**3.93**	**3.79**	**3.68**	**3.59**	**3.52**	**3.45**	**3.35**	**3.27**	**3.16**
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.29	2.25	2.19
	8.28	**6.01**	**5.09**	**4.58**	**4.25**	**4.01**	**3.85**	**3.71**	**3.60**	**3.51**	**3.44**	**3.37**	**3.27**	**3.19**	**3.07**
19	4.38	3.52	3.13	2.90	2.74	2.63	2.55	2.48	2.43	2.38	2.34	2.31	2.26	2.21	2.15
	8.18	**5.93**	**5.01**	**4.50**	**4.17**	**3.94**	**3.77**	**3.63**	**3.52**	**3.43**	**3.36**	**3.30**	**3.19**	**3.12**	**3.00**
20	4.35	3.49	3.10	2.87	2.71	2.60	2.52	2.45	2.40	2.35	2.31	2.28	2.23	2.18	2.12
	8.10	**5.85**	**4.94**	**4.43**	**4.10**	**3.87**	**3.71**	**3.56**	**3.45**	**3.37**	**3.30**	**3.23**	**3.13**	**3.05**	**2.94**
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25	2.20	2.15	2.09
	8.02	**5.78**	**4.87**	**4.37**	**4.04**	**3.81**	**3.65**	**3.51**	**3.40**	**3.31**	**3.24**	**3.17**	**3.07**	**2.99**	**2.88**
22	4.30	3.44	3.05	2.82	2.66	2.55	2.47	2.40	2.35	2.30	2.26	2.23	2.18	2.13	2.07
	7.94	**5.72**	**4.82**	**4.31**	**3.99**	**3.76**	**3.59**	**3.45**	**3.35**	**3.26**	**3.18**	**3.12**	**3.02**	**2.94**	**2.83**
23	4.28	3.42	3.03	2.80	2.64	2.53	2.45	2.38	2.32	2.28	2.24	2.20	2.14	2.10	2.04
	7.88	**5.66**	**4.76**	**4.26**	**3.94**	**3.71**	**3.54**	**3.41**	**3.30**	**3.21**	**3.14**	**3.07**	**2.97**	**2.89**	**2.78**
24	4.26	3.40	3.01	2.78	2.62	2.51	2.43	2.36	2.30	2.26	2.22	2.18	2.13	2.09	2.02
	7.82	**5.61**	**4.72**	**4.22**	**3.90**	**3.67**	**3.50**	**3.36**	**3.25**	**3.17**	**3.09**	**3.03**	**2.93**	**2.85**	**2.74**
25	4.24	3.38	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24	2.20	2.16	2.11	2.06	2.00
	7.77	**5.57**	**4.68**	**4.18**	**3.86**	**3.63**	**3.46**	**3.32**	**3.21**	**3.13**	**3.05**	**2.99**	**2.89**	**2.81**	**2.70**
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	2.10	2.05	1.99
	7.72	**5.53**	**4.64**	**4.14**	**3.82**	**3.59**	**3.42**	**3.29**	**3.17**	**3.09**	**3.02**	**2.96**	**2.86**	**2.77**	**2.66**
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.30	2.25	2.20	2.16	2.13	2.08	2.03	1.97
	7.68	**5.49**	**4.60**	**4.11**	**3.79**	**3.56**	**3.39**	**3.26**	**3.14**	**3.06**	**2.98**	**2.93**	**2.83**	**2.74**	**2.63**
28	4.20	3.34	2.95	2.71	2.56	2.44	2.36	2.29	2.24	2.19	2.15	2.12	2.06	2.02	1.96
	7.64	**5.45**	**4.57**	**4.07**	**3.76**	**3.53**	**3.36**	**3.23**	**3.11**	**3.03**	**2.95**	**2.90**	**2.80**	**2.71**	**2.60**
29	4.18	3.33	2.93	2.70	2.54	2.43	2.35	2.28	2.22	2.18	2.14	2.10	2.05	2.00	1.94
	7.60	**5.42**	**4.54**	**4.04**	**3.73**	**3.50**	**3.33**	**3.20**	**3.08**	**3.00**	**2.92**	**2.87**	**2.77**	**2.68**	**2.57**
30	4.17	3.32	2.92	2.69	2.53	2.42	2.34	2.27	2.21	2.16	2.12	2.09	2.04	1.99	1.93
	7.56	**5.39**	**4.51**	**4.02**	**3.70**	**3.47**	**3.30**	**3.17**	**3.06**	**2.98**	**2.90**	**2.84**	**2.74**	**2.66**	**2.55**
32	4.15	3.30	2.90	2.67	2.51	2.40	2.32	2.25	2.19	2.14	2.10	2.07	2.02	1.97	1.91
	7.50	**5.34**	**4.46**	**3.97**	**3.66**	**3.42**	**3.25**	**3.12**	**3.01**	**2.94**	**2.86**	**2.80**	**2.70**	**2.62**	**2.51**
34	4.13	3.28	2.88	2.65	2.49	2.38	2.30	2.23	2.17	2.12	2.08	2.05	2.00	1.95	1.89
	7.44	**5.29**	**4.42**	**3.93**	**3.61**	**3.38**	**3.21**	**3.08**	**2.97**	**2.89**	**2.82**	**2.76**	**2.66**	**2.58**	**2.47**
36	4.11	3.26	2.86	2.63	2.48	2.36	2.28	2.21	2.15	2.10	2.06	2.03	1.98	1.93	1.87
	7.39	**5.25**	**4.38**	**3.89**	**3.58**	**3.35**	**3.18**	**3.04**	**2.94**	**2.86**	**2.78**	**2.72**	**2.62**	**2.54**	**2.43**
38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09	2.05	2.02	1.96	1.92	1.85
	7.35	**5.21**	**4.34**	**3.86**	**3.54**	**3.32**	**3.15**	**3.02**	**2.91**	**2.82**	**2.75**	**2.69**	**2.59**	**2.51**	**2.40**
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.07	2.04	2.00	1.95	1.90	1.84
	7.31	**5.18**	**4.31**	**3.83**	**3.51**	**3.29**	**3.12**	**2.99**	**2.88**	**2.80**	**2.73**	**2.66**	**2.56**	**2.49**	**2.37**