## The *t* statistic

**The *t* statistic** is used to test hypotheses about an unknown population mean, μ , when the value of σ is unknown

## The estimated standard error

The **estimated standard error** ($s_M$) is used as an estimate of the real standard error, $s_M$, when the value of σ is unknown. It is computed using the sample variance or sample standard deviation and provides an estimate of the standard distance between a sample mean, *M*, and the population mean, μ.

## Degrees of freedom

**Degrees of freedom** describe the number of scores in a sample that are independent and free to vary. Because the sample mean places a restriction on the value of one score in the sample, there are *n* – 1 degrees of freedom for a sample with *n* scores

## *t* distribution

**A *t*** distribution is the complete set of *t* values computed for every possible random sample for a specific sample size (*n*) or a specific degrees of freedom (*df*). The *t* distribution approximates the shape of a normal distribution, especially for large samples or samples from a normal population.

## Assumptions of the t Test

Two basic assumptions are necessary for hypothesis tests with the *t* statistic.

**1**. **The values in the sample must consist of *independent* observations.**
In everyday terms, two observations are independent if there is no consistent, predictable relationship between the first observation and the second. More precisely, two events (or observations) are independent if the occurrence of the first event has no effect on the probability of the second event.

**2. The population that is sampled must be normal.**
This assumption is a necessary part of the mathematics underlying the development of the *t* statistic and the *t* distribution table. However, violating this assumption has little practical effect on the results obtained for a *t* statistic, especially when the sample size is relatively large. With very small samples, a normal population distribution is important. With larger samples, this assumption can be violated without affecting the validity of the hypothesis test. If you have reason to suspect that the population distribution is not normal, use a large sample to be safe.

**Example: A Hypothesis Test with the t Statistic**

A psychologist has prepared an "Optimism Test" that is administered yearly to graduating college seniors. The test measures how each graduating class feels about its future—the higher the score, the more optimistic the class. Last year's class had a mean score of $\mu=15$. A sample of $n = 9$ seniors from this year's class was selected and tested. The scores for these seniors are 7, 12, 11, 15, 7, 8, 15, 9, and 6, which produce a sample mean of $M = 10$ with $SS=94$. On the basis of this sample, can the psychologist conclude that this year's class has a different level of optimism than last year's class?
Note that this hypothesis test uses a $t$ statistic because the population variance ($\sigma^2$) is not known.
For this demonstration, we use a $\alpha= .05$, two tails.

**State the hypotheses**

The null hypothesis states that the mean optimism score for this year's class is the same as the mean for last year's class.
$H_0$: $\mu=15$ (There is no change.)
$H_1$: $\mu\neq15$ (This year's mean is different.)

**Locate the critical region**

With a sample of $n = 9$ students, the $t$ statistic has $df = n - 1 = 8$. For a two-tailed test with $\alpha= .05$ and $df = 8$, the critical $t$ values are $t = \pm2.306$. These critical $t$ values define the boundaries of the critical region.

**Compute the test statistic**

As we have noted, it is easier to separate the calculation of the $t$ statistic into three stages.
*Sample variance*.

$$s^2 = \frac{SS}{df} = \frac{SS}{n-1} = \frac{94}{8} = 11.75$$

*Estimated standard error*. The estimated standard error for these data is

$$s_M = \frac{s}{\sqrt{n}} \; or \; s_M = \sqrt{\frac{s^2}{n}} = \sqrt{\frac{11.75}{9}} = 1.14$$

*The t statistic*. Now that we have the estimated standard error and the sample mean, we can compute the $t$ statistic. For this demonstration,

$$t = \frac{M-\mu}{s_M} = \frac{10-15}{1.14} = -4.39$$

**Make a decision about $H_0$, and state a conclusion.**

The $t$ statistic we obtained ($t = -4.39$) is in the critical region. Thus, our sample data are unusual enough to reject the null hypothesis at the .05 level of significance. We can conclude that there is a significant difference in level of optimism between this year's and last year's graduating classes, $t(8) = -4.39$, $p < .05$, two-tailed.

***Effect Size: Estimating Cohen's d and Computing $r^2$***

We estimate Cohen's *d* for the same data used for the hypothesis test in demonstration. The mean optimism score for the sample from this year's class was 5 points lower than the mean from last year ($M = 10$ versus $\mu=15$).

In demonstration, we computed a sample variance of $s^2 = 11.75$, so the standard deviation is $s = \sqrt{11.75} = 3.4$ .With these values,

$$\text{estimated d} = \frac{\text{mean difference}}{\text{sample standard deviation}} = \frac{M - \mu}{s}$$

To calculate the percentage of variance explained by the treatment effect, $r^2$, we need the value of $t$ and the $df$ value from the hypothesis test. In demonstration, we obtained $t = -4.39$ with $df = 8$. Using these values in the below equation, we obtain

$$r^2 = \frac{t^2}{t^2 + df} = \frac{(-4.39)^2}{(-4.39)^2 + 8} = 0.71$$

## Exercises

1. A sample of $n = 16$ individuals is selected from a population with a mean of $\mu = 80$. A treatment is administered to the sample and, after treatment, the sample mean is found to be $M=86$ with a standard deviation of $s = 8$.
a. Does the sample provide sufficient evidence to conclude that the treatment has a significant effect? Test with $\alpha = .05$
b. Compute Cohen's $d$ and $r^2$ to measure the effect size.
c. Find the 95% confidence interval for the population mean after treatment.

2. How does sample size influence the outcome of a hypothesis test and measures of effect size? How does the standard deviation influence the outcome of a hypothesis test and measures of effect size?

**3.** If all other factors are held constant, an 80% confidence interval is wider than a 90% confidence interval. (True or false?)
**4.** If all other factors are held constant, a confidence interval computed from a sample of $n = 25$ is wider than a confidence interval computed from a sample of $n = 100$. (True or false?)

## Answers

1. a. The estimated standard error is 2 points and the data produce $t = \frac{6}{2} = 3$. With *df* =15, the critical values are *t* = ± 2.131, so the decision is to reject $H_0$ and conclude that there is a significant treatment effect.
b. For these data, $d = \frac{6}{8} = 0.75$ and $r^2 = \frac{9}{24} = 0.375$ or 37.5%
c. For 95% confidence and *df* = 15, use *t* = ± 2.131. The confidence interval is μ=86±2(2.131) and extends from 81.738 to 90.262.

2. Increasing sample size increases the likelihood of rejecting the null hypothesis but has little or no effect on measures of effect size. Increasing the sample variance reduces the likelihood of rejecting the null hypothesis and reduces measures of effect size.

3. False. Greater confidence requires a wider interval.

4. True. The smaller sample produces a wider interval.

## QUESTIONS

1. A developmental psychologist would like to know whether success in one specific area can affect a person's self-esteem. The psychologist selects a sample of n=25 10-year-old children who all excel in athletics. These children are given a standardized self-esteem test for which the general population of 10-year-old children averages μ=70. The average score for the sample is M=74.5 with SS=2400. On the basis of these data, can the psychologist conclude that excelling in athletics has a general effect on self-esteem? Use a two-tailed with α=.05.

2. A college professor has noted that this year's freshman class appears to be smarter than classes from previous years. The professor obtains a sample of n=36 freshmen and computes an average IQ of M=114.5 with a variance of 324 for this group. College records indicate that the mean IQ for entering freshman from the earlier years is μ=110.3. On the basis of these data, can the professor conclude that this year's students have IQs that are significantly different from those of previous students? Use a two-tailed with α=.05.

3. Educational administers have complained for years that American high school students are dismally ignorant about world geography. To evaluate this complaint, a history teacher prepared a 40-question multiple-choice geography test. Each question had four choices for the answer, so the probability of guessing correctly is p=1/4. Thus chance performance would result in an average score of μ=10 out of 40. The test was administered to a random sample of n=36 students, and mean score for the sample was M=13.5 with variance=144. On the basis of these data, can the teacher conclude that the student's score are significantly different from what would be expected by chance? Use a two-tailed with α=.05.

4. The personnel department for a major corporation in the Northeast reported that the average number of absences during months of January and February last year was μ=7.4. In an attempt to reduce absences, the company offered a free flu shots to all employees this year. For a sample of n=100 people who took the shots, the average number of

absences this year was $M=4.2$ with $SS=396$. Do these data indicate a significant reduction in the number of absences? Use a one-tailed test with $\alpha=.05$.

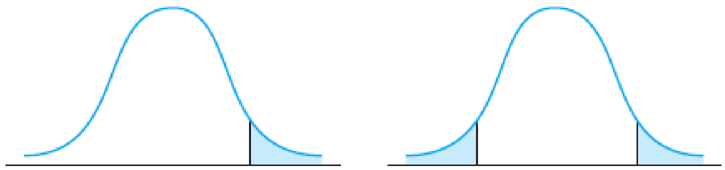
5. The newspaper article reported that the typical American family spent an average of $\mu=\$81$ for Halloween candy and costumes last year. A sample of $n=16$ families this year produced a mean of $M=\$85$ with $SS=6000$. Do these data indicate a significant increase in holiday spending? Use a one-tailed test with $\alpha=.01$.

6. What factor determines whether you should use a z-score or a t statistic for a hypothesis test?

7. For each of the following, describe how the value of t is affected. In each case, assume that all other factors are held constant.
a. What happens to the value of t when the variability of the scores in the sample increases?
b. What happens to the value of t when the number of the scores in the sample increases?
c. What happens to the value of t when the difference between the sample mean and the hypothesized population mean increases?

## TABLE B.2 THE *t* DISTRIBUTION

Table entries are values of *t* corresponding to proportions in one tail or in two tails combined.



One tail (either right or left)

Two tails combined

| | Proportion in One Tail | | | | | |
|---|---|---|---|---|---|---|
| | 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| | Proportion in Two Tails Combined | | | | | |
| df | 0.50 | 0.20 | 0.10 | 0.05 | 0.02 | 0.01 |
| 1 | 1.000 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 0.816 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 0.765 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 0.741 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 0.727 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 0.718 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 0.711 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 0.706 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 0.703 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 0.700 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 0.697 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 0.695 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 0.694 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 0.692 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 0.691 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | 0.690 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 0.689 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 0.688 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 0.688 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 0.687 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | 0.686 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 0.686 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 0.685 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 0.685 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 0.684 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | 0.684 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | 0.684 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 0.683 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 0.683 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | 0.683 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 40 | 0.681 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 60 | 0.679 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 120 | 0.677 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 |
| ∞ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |

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