TWO FACTOR ANOVA

14.2 MAIN EFFECTS AND INTERACTIONS

As noted in the previous section, a two-factor ANOVA actually involves three distinct hypothesis tests. In this section, we examine these three tests in more detail.

Traditionally, the two independent variables in a two-factor experiment are identified as factor *A* and factor *B*. For the study presented in Table 14.1, self-esteem is factor *A*, and the presence or absence of an audience is factor B. The goal of the study is to evaluate the mean differences that may be produced by either of these factors acting independently or by the two factors acting together.

MAIN EFFECTS

One purpose of the study is to determine whether differences in self-esteem (factor *A*) result in differences in performance. To answer this question, we compare the mean score for all of the participants with low self-esteem with the mean for those with high self-esteem. Note that this process evaluates the mean difference between the top row and the bottom row in Table 14.1.

To make this process more concrete, we present a set of hypothetical data in Table 14.2. The table shows the mean score for each of the treatment conditions (cells) as well as the overall mean for each column (each audience condition) and the overall mean for each row (each self-esteem group). These data indicate that the low self-esteem participants (the top row) had an overall mean of $M = 8$ errors. This overall mean was obtained by computing the average of the two means in the top row. In

	No Audience	Audience	
Low	$M = 7$	$M = 9$	$M = 8$
High	$M = 3$	$M = 5$	$M = 4$
	$M = 5$	$M = 7$	

contrast, the high self-esteem participants had an overall mean of $M = 4$ errors (the mean for the bottom row). The difference between these means constitutes what is called the *main effect* for self-esteem, or the *main effect for factor* A.

Similarly, the main effect for factor *B* (audience condition) is defined by the mean difference between the columns of the matrix. For the data in Table 14.2, the two groups of participants tested with no audience had an overall mean score of $M = 5$ errors. Participants tested with an audience committed an overall average of $M = 7$ errors. The difference between these means constitutes the *main effect* for the audience conditions, or the *main effect for factor* B.

TION

The mean differences among the levels of one factor are referred to as the **main effect** of that factor. When the design of the research study is represented as a matrix with one factor determining the rows and the second factor determining the columns, then the mean differences among the rows describe the main effect of one factor, and the mean differences among the columns describe the main effect for the second factor.

For the example we are considering, factor *A* involves the comparison of two different levels of self-esteem. The null hypothesis would state that there is no difference between the two levels; that is, self-esteem has no effect on performance. In symbols,

$$H_0: \quad \mu_{A_1} = \mu_{A_2}$$

The alternative hypothesis is that the two different levels of self-esteem do produce different scores:

$$H_1: \quad \mu_{A_1} \neq \mu_{A_2}$$

To evaluate these hypotheses, we compute an *F*-ratio that compares the actual mean differences between the two self-esteem levels versus the amount of difference that would be expected without any systematic treatment effects.

$$F = \frac{\text{variance (differences) between the means for factor } A}{\text{variance (differences) expected if there is no treatment effect}}$$

$$F = \frac{\text{variance (differences) between the row means}}{\text{variance (differences) expected if there is no treatment effect}}$$

Similarly, factor *B* involves the comparison of the two different audience conditions. The null hypothesis states that there is no difference in the mean number of errors between the two conditions. In symbols,

$$H_0: \quad \mu_{B_1} = \mu_{B_2}$$

As always, the alternative hypothesis states that the means are different:

$$H_1: \quad \mu_{B_1} \neq \mu_{B_2}$$

Again, the *F*-ratio compares the obtained mean difference between the two audience conditions versus the amount of difference that would be expected if there is no systematic treatment effect.

$$F = \frac{\text{variance (differences) between the means for factor } B}{\text{variance (differences) expected if there is no treatment effect}}$$

$$F = \frac{\text{variance (differences) between the column means}}{\text{variance (differences) expected if there is no treatment effect}}$$

INTERACTIONS In addition to evaluating the main effect of each factor individually, the two-factor ANOVA allows you to evaluate other mean differences that may result from unique combinations of the two factors. For example, specific combinations of self-esteem and an audience acting together may have effects that are different from the effects of self-esteem or an audience acting alone. Any "extra" mean differences that are not explained by the main effects are called an *interaction,* or an *interaction between factors*. The real advantage of combining two factors within the same study is the ability to examine the unique effects caused by an interaction.

DEFINITION

An **interaction** between two factors occurs whenever the mean differences between individual treatment conditions, or cells, are different from what would be predicted from the overall main effects of the factors.

To make the concept of an interaction more concrete, we reexamine the data shown in Table 14.2. For these data, there is no interaction; that is, there are no extra mean differences that are not explained by the main effects. For example, within each audience condition (each column of the matrix) the average number of errors for the low self-esteem participants is 4 points higher than the average for the high self-esteem participants. This 4-point mean difference is exactly what is predicted by the overall main effect for self-esteem.

Now consider a different set of data shown in Table 14.3. These new data show exactly the same main effects that existed in Table 14.2 (the column means and the row

$$F = \frac{\text{variance (mean differences) not explained by main effects}}{\text{variance (differences) expected if there is no treatment effects}}$$

The null hypothesis for this F-ratio simply states that there is no interaction:

H_0: There is no interaction between factors A and B. All of the mean differences between treatment conditions are explained by the main effects of the two factors.

The alternative hypothesis is that there is an interaction between the two factors:

H_1: There is an interaction between factors. The mean differences between treatment conditions are not what would be predicted from the overall main effects of the two factors.

TION

When the effect of one factor depends on the different levels of a second factor, then there is an **interaction** between the factors.

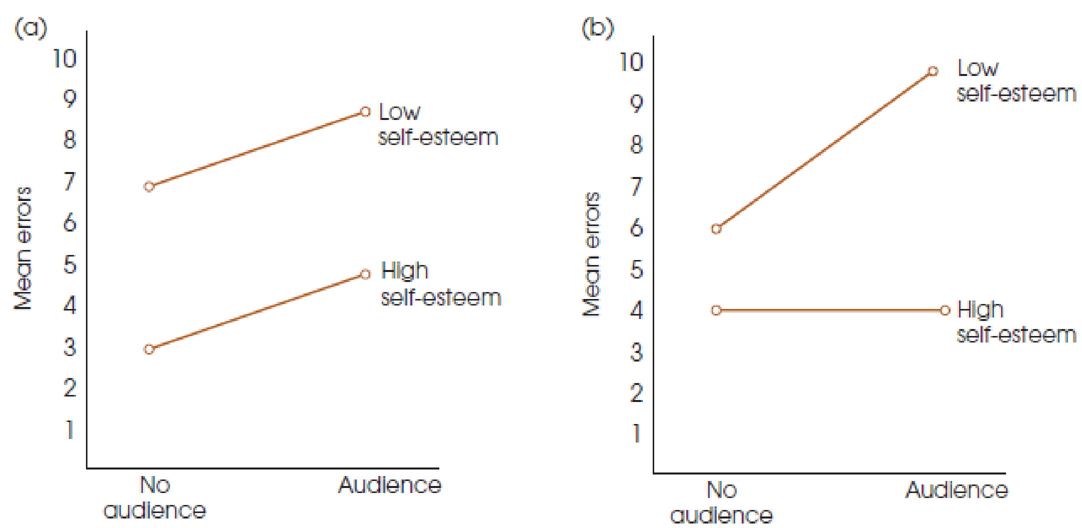


FIGURE 14.2

(a) Graph showing the treatment means from Table 14.2, for which there is no reaction. (b) Graph for Table 14.3, for which there is an interaction.

I O N

When the results of a two-factor study are presented in a graph, the existence of nonparallel lines (lines that cross or converge) indicates an **interaction** between the two factors.

BOX 14.1 GRAPHING RESULTS FROM A TWO-FACTOR DESIGN

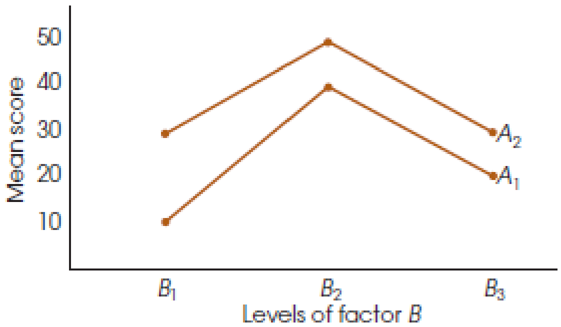
One of the best ways to get a quick overview of the results from a two-factor study is to present the data in a line graph. Because the graph must display the means obtained for *two* independent variables (two factors), constructing the graph can be a bit more complicated than constructing the single-factor graphs we presented in Chapter 3 (pp. 93–95).

Figure 14.3 shows a line graph presenting the results from a two-factor study with 2 levels of factor *A* and 3 levels of factor *B*. With a 2 × 3 design, there are a total of 6 different treatment means, which are shown in the following matrix.

		Factor *B*		
		B1	*B2*	*B3*
Factor *A*	A_1	10	40	20
	A_2	30	50	30

In the graph, note that values for the dependent variable (the treatment means) are shown on the vertical axis. Also note that the levels for one factor (we selected factor *B*) are displayed on the horizontal axis. Directly above the B_1 value on the horizontal axis, we have placed two dots corresponding to the two means in the B_1 column of the data matrix. Similarly, we have placed two dots above B_2 and another two dots above B_3. Finally, we have drawn a line connecting the three dots corresponding to level 1 of factor *A* (the three means in the top row of the data matrix). We have also drawn a second line that connects the three dots corresponding to level 2 of factor *A*. These lines are labeled A_1 and A_2 in the figure.

FIGURE 14.3
A line graph showing the results from a two-factor experiment.



(a) Data showing a main effect for factor *A* but no *B* effect and no interaction

	B_1	B_2	
A_1	20	20	A_1 mean = 20
A_2	10	10	A_2 mean = 10
	B_1 mean = 15	B_2 mean = 15	

10-point difference

No difference

(b) Data showing main effects for both factor A and factor B but no interaction

	B_1	B_2	
A_1	10	30	A_1 mean = 20
A_2	20	40	A_2 mean = 30
	B_1 mean = 15	B_2 mean = 35	

10-point difference

20-point difference

(c) Data showing no main effect for either factor but an interaction

	B_1	B_2	
A_1	10	20	A_1 mean = 15
A_2	20	10	A_2 mean = 15
	B_1 mean = 15	B_2 mean = 15	

No difference

No difference

CHECK

1. Each of the following matrices represents a possible outcome of a two-factor experiment. For each experiment:
   a. Describe the main effect for factor A.
   b. Describe the main effect for factor B.
   c. Does there appear to be an interaction between the two factors?

Experiment I

	B_1	B_2
A_1	$M = 10$	$M = 20$
A_2	$M = 30$	$M = 40$

Experiment II

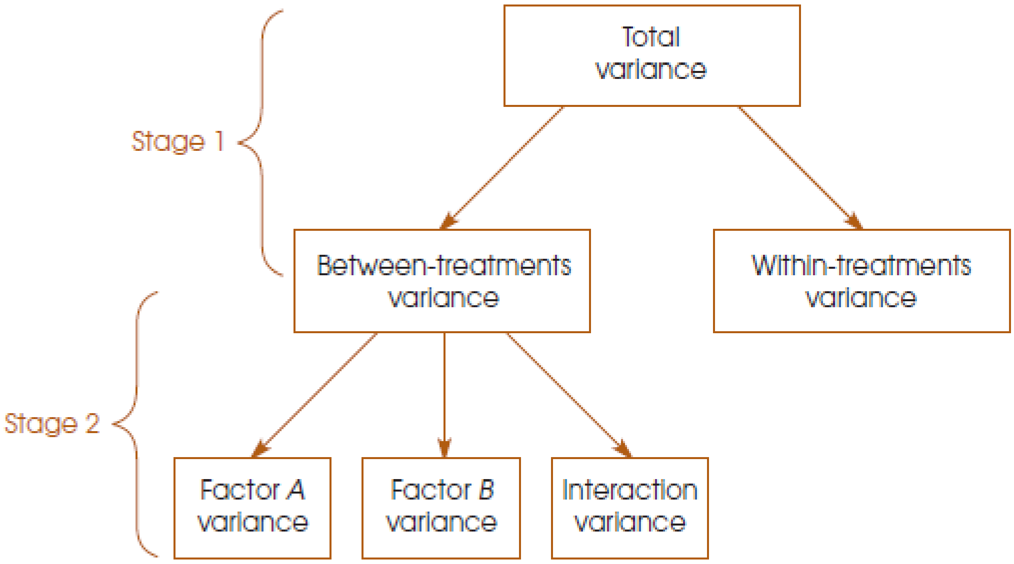
	B_1	B_2
A_1	$M = 10$	$M = 30$
A_2	$M = 20$	$M = 20$

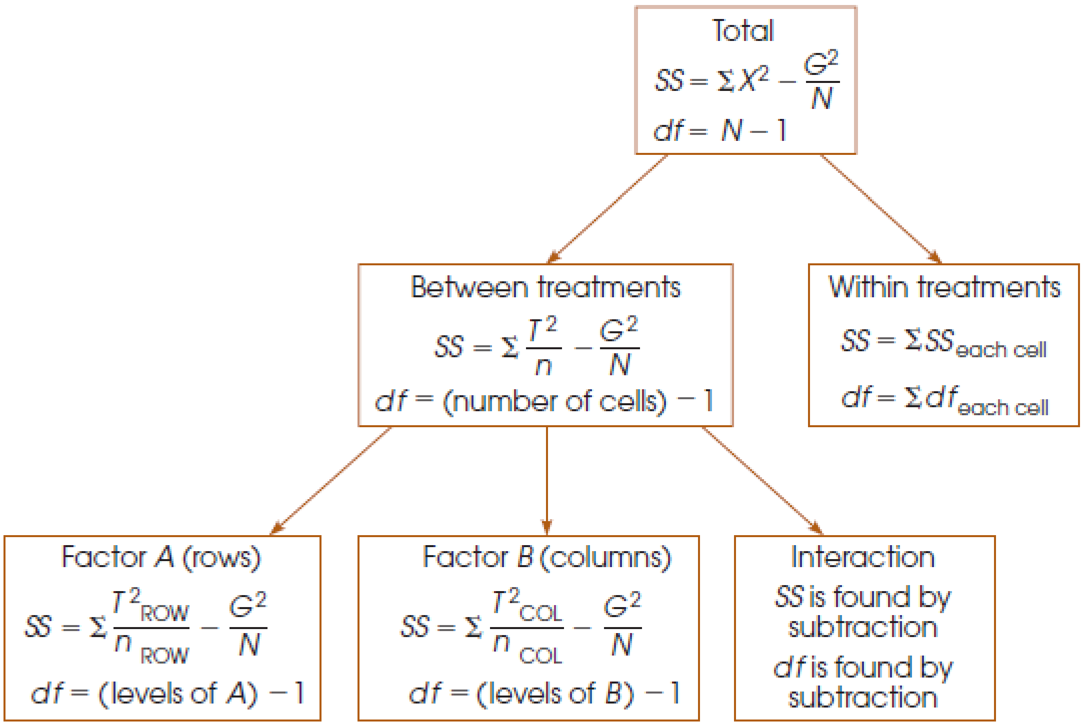
2. In a graph showing the means from a two-factor experiment, parallel lines indicate that there is no interaction. (True or false?)
3. A two-factor ANOVA consists of three hypothesis tests. What are they?
4. It is impossible to have an interaction unless you also have main effects for at least one of the two factors. (True or false?)

ANSWERS

1. For Experiment I:
   a. There is a main effect for factor A; the scores in A_2 average 20 points higher than in A_1.
   b. There is a main effect for factor B; the scores in B_2 average 10 points higher than in B_1.
   c. There is no interaction; there is a constant 20-point difference between A_1 and A_2 that does not depend on the levels of factor B.

   For Experiment II:
   a. There is no main effect for factor A; the scores in A_1 and in A_2 both average 20.
   b. There is a main effect for factor B; on average, the scores in B_2 are 10 points higher than in B_1.
   c. There is an interaction. The difference between A_1 and A_2 depends on the level of factor B. (There is a +10 difference in B_1 and a −10 difference in B_2.)
2. True.
3. The two-factor ANOVA evaluates the main effect for factor A, the main effect for factor B, and the interaction between the two factors.
4. False. Main effects and interactions are completely independent.





$$MS_{\text{factor}} = \frac{SS \text{ for the factor}}{df \text{ for the factor}} \qquad MS_{\text{within}} = \frac{SS_{\text{within treatments}}}{df_{\text{within treatments}}}$$

14.1 We use the data shown in Table 14.5 to demonstrate the two-factor ANOVA. The data are representative of many studies examining the relationship between arousal and performance. The general result of these studies is that increasing the level of arousal (or motivation) tends to improve the level of performance. (You probably have tried to "psych yourself up" to do well on a task.) For very difficult tasks, however, increasing arousal beyond a certain point tends to lower the level of performance. (Your friends have probably advised you to "calm down and stay focused" when you get overanxious about doing well.) This relationship between arousal and performance is known as the Yerkes-Dodson law.

The data are displayed in a matrix with the two levels of task difficulty (factor *A*) making up the rows and the three levels of arousal (factor *B*) making up

		Factor *B* Arousal Level				
		Low	Medium	High		
Factor *A* Task Difficulty	Easy	3 1 1 6 4 $M = 3$ $T = 15$ $SS = 18$	1 4 8 6 6 $M = 5$ $T = 25$ $SS = 28$	10 10 14 7 9 $M = 10$ $T = 50$ $SS = 26$	$T_{ROW1} = 90$	$N = 30$ $G = 120$ $\Sigma X^2 = 860$
	Difficult	0 2 0 0 3 $M = 1$ $T = 5$ $SS = 8$	2 2 2 2 $M = 3$ $T = 15$ $SS = 20$	1 7q 1 6 1 $M = 2$ $T = 10$ $SS = 20$	$T_{ROW2} = 30$	
		$T_{COL1} = 20$	$T_{COL2} = 40$	$T_{COL3} = 60$		

the columns. For the easy task, note that performance scores increase consistently as arousal increases. For the difficult task, on the other hand, performance peaks at a medium level of arousal and drops when arousal is increased to a high level. Note that the data matrix has a total of six *cells,* or treatment conditions, with a separate sample of $n = 5$ subjects in each condition. Most of the notation should be familiar from the single-factor ANOVA presented in Chapter 12. Specifically, the treatment totals are identified by T values, the total number of scores in the entire study is $N = 30$, and the grand total (sum) of all 30 scores is $G = 120$. In addition to these familiar values, we have included the totals for each row and for each column in the matrix. The goal of the ANOVA is to determine whether the mean differences observed in the data are significantly greater than would be expected if there are no treatment effects.

The first stage of the two-factor ANOVA separates the total variability into two components: between-treatments and within-treatments. The formulas for this stage are identical to the formulas used in the single-factor ANOVA in Chapter 12 with the provision that each cell in the two-factor matrix is treated as a separate treatment condition. The formulas and the calculations for the data in Table 14.5 are as follows:

Total variability

$$SS_{total} = \Sigma X^2 - \frac{G^2}{N} \qquad (14.2)$$

For these data,

$$SS_{\text{total}} = 860 - \frac{120^2}{30}$$

$$= 860 - 480$$

$$= 380$$

This SS value measures the variability for all $N = 30$ scores and has degrees of freedom given by

$$df_{\text{total}} = N - 1 \quad (14.3)$$

For the data in Table 14.5, $df_{\text{total}} = 29$.

Within-treatments variability To compute the variance within treatments, we first compute SS and $df = n - 1$ for each of the individual treatment conditions. Then the within-treatments SS is defined as

$$SS_{\text{within treatments}} = \Sigma SS_{\text{each treatment}} \quad (14.4)$$

And the within-treatments df is defined as

$$df_{\text{within treatments}} = \Sigma df_{\text{each treatment}} \quad (14.5)$$

For the six treatment conditions in Table 14.4,

$$SS_{\text{within treatments}} = 18 + 28 + 26 + 8 + 20 + 20$$

$$= 120$$

$$df_{\text{within treatments}} = 4 + 4 + 4 + 4 + 4 + 4$$

$$= 24$$

Between-treatments variability Because the two components in stage 1 must add up to the total, the easiest way to find $SS_{\text{between treatments}}$ is by subtraction.

$$SS_{\text{between treatments}} = SS_{\text{total}} - SS_{\text{within}} \quad (14.6)$$

For the data in Table 14.4, we obtain

$$SS_{\text{between treatments}} = 380 - 120 = 260$$

However, you can also use the computational formula to calculate $SS_{\text{between treatments}}$ directly.

$$SS_{\text{between treatments}} = \Sigma\frac{T^2}{n} - \frac{G^2}{N} \quad (14.7)$$

For the data in Table 14.4, there are six treatments (six T values), each with $n = 5$ scores, and the between-treatments SS is

$$SS_{\text{between treatments}} = \frac{15^2}{5} + \frac{25^2}{5} + \frac{50^2}{5} + \frac{5^2}{5} + \frac{15^2}{5} + \frac{10^2}{5} - \frac{120^2}{30}$$

$$= 45 + 125 + 500 + 5 + 45 + 20 - 480$$

$$= 260$$

The between-treatments df value is determined by the number of treatments (or the number of T values) minus one. For a two-factor study, the number of treatments is equal to the number of cells in the matrix. Thus,

$$df_{\text{between treatments}} = \text{number of cells} - 1 \qquad (14.8)$$

For these data, $df_{\text{between treatments}} = 5$.

This completes the first stage of the analysis. Note that the two components, when added, equal the total for both SS values and df values.

$$SS_{\text{between treatments}} + SS_{\text{within treatments}} = SS_{\text{total}}$$

$$240 + 120 = 360$$

$$df_{\text{between treatments}} + df_{\text{within treatments}} = df_{\text{total}}$$

$$5 + 24 = 29$$

The second stage of the analysis determines the numerators for the three F-ratios. Specifically, this stage determines the between-treatments variance for factor A, factor B, and the interaction.

1. Factor A. The main effect for factor A evaluates the mean differences between the levels of factor A. For this example, factor A defines the rows of the matrix, so we are evaluating the mean differences between rows. To compute the SS for factor A, we calculate a between-treatment SS using the row totals in exactly the same way that we computed $SS_{\text{between treatments}}$ using the treatment totals (T values) earlier. For factor A, the row totals are 90 and 30, and each total was obtained by adding 15 scores.

Therefore,

$$SS_A = \Sigma \frac{T^2_{ROW}}{n_{ROW}} - \frac{G^2}{N} \qquad (14.9)$$

For our data,

$$SS_A = \frac{90^2}{15} + \frac{30^2}{15} - \frac{120^2}{30}$$

$$= 540 + 60 - 480$$

$$= 120$$

Factor A involves two treatments (or two rows), easy and difficult, so the df value is

$$df_A = \text{number of rows} - 1 \qquad (14.10)$$

$$= 2 - 1$$

$$= 1$$

2. Factor B. The calculations for factor B follow exactly the same pattern that was used for factor A, except for substituting columns in place of rows. The main effect for factor *B* evaluates the mean differences between the levels of factor *B*, which define the columns of the matrix.

$$SS_B = \Sigma\frac{T_{COL}^2}{n_{COL}} - \frac{G^2}{N} \qquad (14.11)$$

For our data, the column totals are 20, 40, and 60, and each total was obtained by adding 10 scores. Thus,

$$\begin{aligned} SS_B &= \frac{20^2}{10}+\frac{40^2}{10}+\frac{60^2}{10}-\frac{120^2}{30} \\ &= 40+160+360-480 \\ &= 80 \end{aligned}$$

$$\begin{aligned} df_B &= \text{number of columns} - 1 \qquad (14.12) \\ &= 3 - 1 \\ &= 2 \end{aligned}$$

3. The $A \times B$ Interaction. The $A \times B$ interaction is defined as the "extra" mean differences not accounted for by the main effects of the two factors. We use this definition to find the *SS* and *df* values for the interaction by simple subtraction. Specifically, the between-treatments variability is partitioned into three parts: the *A* effect, the *B* effect, and the interaction (see Figure 14.4). We have already computed the *SS* and *df* values for *A* and *B,* so we can find the interaction values by subtracting to find out how much is left. Thus,

$$SS_{A\times B} = SS_{\text{between treatments}} - SS_A - SS_B \qquad (14.13)$$

For our data,

$$\begin{aligned} SS_{A\times B} &= 260 - 120 - 80 \\ &= 60 \end{aligned}$$

Similarly,

$$\begin{aligned} df_{A\times B} &= df_{\text{between treatments}} - df_A - df_B \qquad (14.14) \\ &= 5 - 1 - 2 \\ &= 2 \end{aligned}$$

The two-factor ANOVA consists of three separate hypothesis tests with three separate *F*-ratios. The denominator for each *F*-ratio is intended to measure the variance (differences) that would be expected if there are no treatment effects. As we saw in Chapter 12, the within-treatments variance is the appropriate denominator for an independent-measures design. Remember that inside each treatment all of the individuals are treated exactly the same, which means that the differences that exist were not caused by any systematic treatment effects (see Chapter 12, p. 393). The within-treatments variance is called a *mean square,* or *MS*, and is computed as follows:

$$MS_{\text{within treatments}} = \frac{SS_{\text{within treatments}}}{df_{\text{within treatments}}}$$

For the data in Table 14.4,

$$MS_{\text{within treatments}} = \frac{120}{24} = 5.00$$

This value forms the denominator for all three F-ratios.

The numerators of the three F-ratios all measured variance or differences between treatments: differences between levels of factor A, differences between levels of factor B, and extra differences that are attributed to the $A \times B$ interaction. These three variances are computed as follows:

$$MS_A = \frac{SS_A}{df_A} \quad MS_B = \frac{SS_B}{df_B} \quad MS_{A\times B} = \frac{SS_{A\times B}}{df_{A\times B}}$$

For the data in Table 14.5, the three MS values are

$$MS_A = \frac{SS_A}{df_A} = \frac{120}{1} = 120 \qquad MS_B = \frac{SS_B}{df_B} = \frac{80}{2} = 40$$

$$MS_{A\times B} = \frac{SS_{A\times B}}{df_{A\times B}} = \frac{60}{2} = 30$$

Finally, the three F-ratios are

$$F_A = \frac{MS_A}{MS_{\text{within treatments}}} = \frac{120}{5} = 24.00$$

$$F_B = \frac{MS_B}{MS_{\text{within treatments}}} = \frac{40}{5} = 8.00$$

$$F_{A\times B} = \frac{MS_{A\times B}}{MS_{\text{within treatments}}} = \frac{30}{5} = 6.00$$

To determine the significance of each F-ratio, we must consult the F distribution table using the df values for each of the individual F-ratios. For this example, the F-ratio for factor A has $df = 1$ for the numerator and $df = 24$ for the denominator. Checking the table with $df = 1, 24$, we find a critical value of 4.26 for $\alpha = .05$ and a critical value of 7.82 for $\alpha = .01$. Our obtained F-ratio, $F = 24.00$ exceeds both of these values, so we conclude that there is a significant difference between the levels of factor A. That is, performance on the easy task (top row) is significantly different from performance on the difficult task (bottom row).

The F-ratio for factor B has $df = 2, 24$. The critical values obtained from the table are 3.40 for $\alpha = .05$ and 5.61 for $\alpha = .01$. Again, our obtained F-ratio, $F = 8.00$, exceeds both values, so we can conclude that there are significant differences among the levels of factor B. For this study, the three levels of arousal result in significantly different levels of performance.

Finally, the F-ratio for the $A \times B$ interaction has $df = 2, 24$ (the same as factor B). With critical values of 3.40 for $\alpha = .05$ and 5.61 for $\alpha = .01$, our obtained F-ratio of $F = 6.00$ is sufficient to conclude that there is a significant interaction between task difficulty and level of arousal.

Source	SS	df	MS	F
Between treatments	260	5		
Factor A (difficulty)	120	1	120	$F(1, 24) = 24.00$
Factor B (arousal)	80	2	40	$F(2, 24) = 8.00$
$A \times B$	60	2	30	$F(2, 24) = 6.00$
Within treatments	120	24	5	
Total	380	29		

MEASURING EFFECT SIZE FOR THE TWO-FACTOR ANOVA

The general technique for measuring effect size with an ANOVA is to compute a value for η^2, the percentage of variance that is explained by the treatment effects. For a two-factor ANOVA, we compute three separate values for eta squared: one measuring how much of the variance is explained by the main effect for factor *A*, one for factor *B*, and a third for the interaction. As we did with the repeated-measures ANOVA (p. 446), we remove any variability that can be explained by other sources before we calculate the percentage for each of the three specific effects. Thus, for example, before we compute the η^2 for factor *A*, we remove the variability that is explained by factor *B* and the variability explained by the interaction. The resulting equation is,

$$\text{for factor } A,\ \eta^2 = \frac{SS_A}{SS_{total} - SS_B - SS_{A\times B}} \tag{14.15}$$

Note that the denominator of Equation 14.15 consists of the variability that is explained by factor *A* and the other *unexplained* variability. Thus, an equivalent version of the equation is,

$$\text{for factor } A,\ \eta^2 = \frac{SS_A}{SS_A + SS_{\text{within treatments}}} \tag{14.16}$$

Similarly, the η^2 formulas for factor *B* and for the interaction are as follows:

$$\text{for factor } B,\ \eta^2 = \frac{SS_B}{SS_{total} - SS_A - SS_{A\times B}} = \frac{SS_B}{SS_B + SS_{\text{within treatments}}} \tag{14.17}$$

$$\text{for } A \times B,\ \eta^2 = \frac{SS_{A\times B}}{SS_{total} - SS_A - SS_B} = \frac{SS_{A\times B}}{SS_{A\times B} + SS_{\text{within treatments}}} \tag{14.18}$$

Because each of the η^2 equations computes a percentage that is not based on the total variability of the scores, the results are often called *partial* eta squares. For the data in Example 14.1, the equations produce the following values:

$$\eta^2 \text{ for factor } A \text{ (difficulty)} = \frac{120}{380-80-60} = \frac{120}{240} = 0.50\ (50\%)$$

$$\eta^2 \text{ for factor } B \text{ (arousal)} = \frac{80}{380-120-60} = \frac{80}{200} = 0.40\ (40\%)$$

$$\eta^2 \text{ for the interaction} = \frac{60}{380-120-80} = \frac{60}{180} = 0.33\ (33\%)$$

15. The following table summarizes the results from a two-factor study with 3 levels of factor *A* and 3 levels of factor *B* using a separate sample of $n = 9$ participants in each treatment condition. Fill in the missing values. (*Hint:* Start with the *df* values.)

Source	*SS*	*df*	*MS*	
Between treatments	144	____		
Factor *A*	____	____	18	$F =$ ____
Factor *B*	____	____	____	$F =$ ____
$A \times B$ Interaction	____	____	____	$F = 7.0$
Within treatments	____	____	____	
Total	360	____		

15.

Source	*SS*	*df*	*MS*	
Between treatments	144	8		
A	36	2	18	$F(2,72) = 6.00$
B	24	2	12	$F(2,72) = 4.00$
A × B	84	4	21	$F(4,72) = 7.00$
Within treatments	216	72	3	
Total	360	80		

17. The following table summarizes the results from a two-factor study with 2 levels of factor *A* and 3 levels of factor *B* using a separate sample of $n = 11$ participants in each treatment condition. Fill in the missing values. (*Hint:* Start with the *df* values.)

Source	*SS*	*df*	*MS*	
Between treatments	____	____		
Factor *A*	____	____	____	$F = 7$
Factor *B*	____	____	____	$F = 8$
$A \times B$ Interaction	____	____	____	$F = 3$
Within treatments	240	____	____	
Total	____	____		

17.

Source	*SS*	*df*	*MS*	
Between treatments	116	5		
A	28	1	28	$F(1,24) = 7.00$
B	64	2	32	$F(1,24) = 8.00$
A × B	24	2	12	$F(1,24) = 3.00$
Within treatments	240	60	4	
Total	356	65		

5. The following matrix presents the results from an independent-measures, two-factor study with a sample of $n = 10$ participants in each treatment condition. Note that one treatment mean is missing.

		Factor *B*	
		B_1	B_2
Factor *A*	A_1	$M = 20$	$M = 30$
	A_2	$M = 40$	

a. What value for the missing mean would result in no main effect for factor *A*?
b. What value for the missing mean would result in no main effect for factor *B*?
c. What value for the missing mean would result in no interaction?

5. a. $M = 10$
b. $M = 30$
c. $M = 50$

Univariate Analysis of Variance

Between-Subjects Factors

		Value Label	N
difficulty	1	easy	15
	2	difficult	15
aurosal level	1	low	10
	2	medium	10
	3	high	10

Descriptive Statistics

Dependent Variable: score

difficulty	aurosal level	Mean	Std. Deviation	N
easy	low	3.00	2.121	5
	medium	5.00	2.646	5
	high	10.00	2.550	5
	Total	6.00	3.798	15
difficult	low	1.00	1.414	5
	medium	3.00	2.236	5
	high	2.00	2.236	5
	Total	2.00	2.035	15
Total	low	2.00	2.000	10
	medium	4.00	2.539	10
	high	6.00	4.784	10
	Total	4.00	3.620	30

Levene's Test of Equality of Error Variances(a)

Dependent Variable: score

F	df1	df2	Sig.
.185	5	24	.966

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.
a Design: Intercept+difficulty+aurosallevel+difficulty * aurosallevel

Tests of Between-Subjects Effects

Dependent Variable: score

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	260.000(a)	5	52.000	10.400	.000
Intercept	480.000	1	480.000	96.000	.000
difficulty	120.000	1	120.000	24.000	.000
aurosallevel	80.000	2	40.000	8.000	.002
difficulty * aurosallevel	60.000	2	30.000	6.000	.008
Error	120.000	24	5.000		
Total	860.000	30			
Corrected Total	380.000	29			

a R Squared = .684 (Adjusted R Squared = .618)

Post Hoc Tests
aurosal level

Multiple Comparisons

Dependent Variable: score
Scheffe

		Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
(I) aurosal level	(J) aurosal level	Lower Bound	Upper Bound	Lower Bound	Upper Bound	Lower Bound
low	medium	-2.00	1.000	.157	-4.61	.61
	high	-4.00(*)	1.000	.002	-6.61	-1.39
medium	low	2.00	1.000	.157	-.61	4.61
	high	-2.00	1.000	.157	-4.61	.61
high	low	4.00(*)	1.000	.002	1.39	6.61
	medium	2.00	1.000	.157	-.61	4.61

Based on observed means.
* The mean difference is significant at the .05 level.