REPEATED MEASURE ANOVA

REPEATED MEASURE ANOVA

The general purpose of the repeated-measures ANOVA is to determine whether the differences that are found between treatment conditions are significantly greater than would be expected if there is no treatment effect. In the numerator of the *F*-ratio, the *between-treatments variance* measures the actual mean differences between the treatment conditions. The variance in the denominator is intended to measure how much difference is reasonable to expect if there are no systematic treatment effects and no systematic individual differences. In other words, the denominator measures variability caused entirely by random and unsystematic factors. For this reason, the variance in the denominator is called the *error variance*. In this section we examine the elements that make up the two variances in the repeated-measures *F*-ratio.

The numerator of the *F*-ratio: between-treatments variance Logically, any differences that are found between treatments can be explained by only two factors:

1. Systematic Differences Caused by the Treatments. It is possible that the different treatment conditions really do have different effects and, therefore, cause the individuals' scores in one condition to be higher (or lower) than in another. Remember that the purpose for the research study is to determine whether a *treatment effect* exists.

2. Random, Unsystematic Differences. Even if there is no treatment effect, it is possible for the scores in one treatment condition to be different from the scores in another. For example, suppose that I measure your IQ score on a Monday morning. A week later I come back and measure your IQ again under exactly the same conditions. Will you get exactly the same IQ score both times? In fact, minor differences between the two measurement situations would probably cause you to end up with two different scores. For example, for one of the IQ tests you might be more tired, or hungry, or worried, or distracted than you were on the other test. These differences can cause your scores to vary. The same thing can happen in a repeated-measures research study. The same individuals are measured at two or more different times and, even though there may be no difference between the two treatment conditions, you can still end up with different scores. However, these differences are random and unsystematic and are classified as error variance.

Thus, it is possible that any differences (or variance) found between treatments could be caused by treatment effects, and it is possible that the differences could simply be the result of chance. On the other hand, it is *impossible* that the differences between treatments are caused by individual differences. Because the repeatedmeasures design uses exactly the same individuals in every treatment condition, individual differences are *automatically eliminated* from the variance between treatments in the numerator of the *F*-ratio.

The denominator of the *F*-ratio: error variance The goal of the ANOVA is to determine whether the differences that are observed in the data are greater than would be expected without any systematic treatment effects. To accomplish this goal, the denominator of the *F*-ratio is intended to measure how much difference (or variance) is reasonable to expect from random and unsystematic factors. This means that we must measure the variance that exists when there are no treatment effects or any other systematic differences.

We begin exactly as we did with the independent-measures *F*-ratio; specifically,

we calculate the variance that exists within treatments. Recall from Chapter 12 that within each treatment all of the individuals are treated in exactly the same way. Therefore, any differences that exist within treatments cannot be caused by treatment effects.
In a repeated-measures design, however, it is also possible that individual differences can cause systematic differences between the scores within treatments. For example, one individual may score consistently higher than another. To eliminate the
In summary, the *F*-ratio for a repeated-measures ANOVA has the same basic structure as the *F*-ratio for independent measures (Chapter 12) except that it includes no variability caused by individual differences. The individual differences are automatically eliminated from the variance between treatments (numerator) because the repeated-measures design uses the same individuals in all treatments. In the denominator, the individual differences are subtracted during the analysis. As a result, the repeated-measures *F*-ratio has the following structure:

$$F = \frac{\text{between-treatments variance}}{\text{error variance}}$$

$$= \frac{\text{treatment effects + random, unsystematic differences}}{\text{random, unsystematic differences}}$$

Note that this *F*-ratio is structured so that there are no individual differences contributing to either the numerator or the denominator. When there is no treatment effect, the *F*-ratio is balanced because the numerator and denominator are both measuring exactly the same variance. In this case, the *F*-ratio should have a value near 1.00. When research results produce an *F*-ratio near 1.00, we conclude that there is no evidence of a treatment effect and we fail to reject the null hypothesis. On the other hand, when a treatment effect does exist, it contributes only to the numerator and should produce a large value for the *F*-ratio. Thus, a large value for *F* indicates that there is a real treatment effect and, therefore, we should reject the null hypothesis.

EXERCISES

1. Explain why individual differences do not contribute to the between-treatments variability in a repeated-measures study.
2. What sources of variability contribute to the within-treatment variability for a repeated-measures study?
3. Describe the structure of the *F*-ratio for the repeated-measures ANOVA.

ANSWERS

1. Because the individuals in one treatment are exactly the same as the individuals in every other treatment, there are no individual differences from one treatment to another.
2. Variability (differences) within treatments is caused by individual differences and random, unsystematic differences.
3. The numerator of the *F*-ratio measures between-treatments variability, which consists of treatment effects and random, unsystematic differences. The denominator measures variability that is exclusively caused by random, unsystematic differences.

REPEATED MEASURE ANOVA

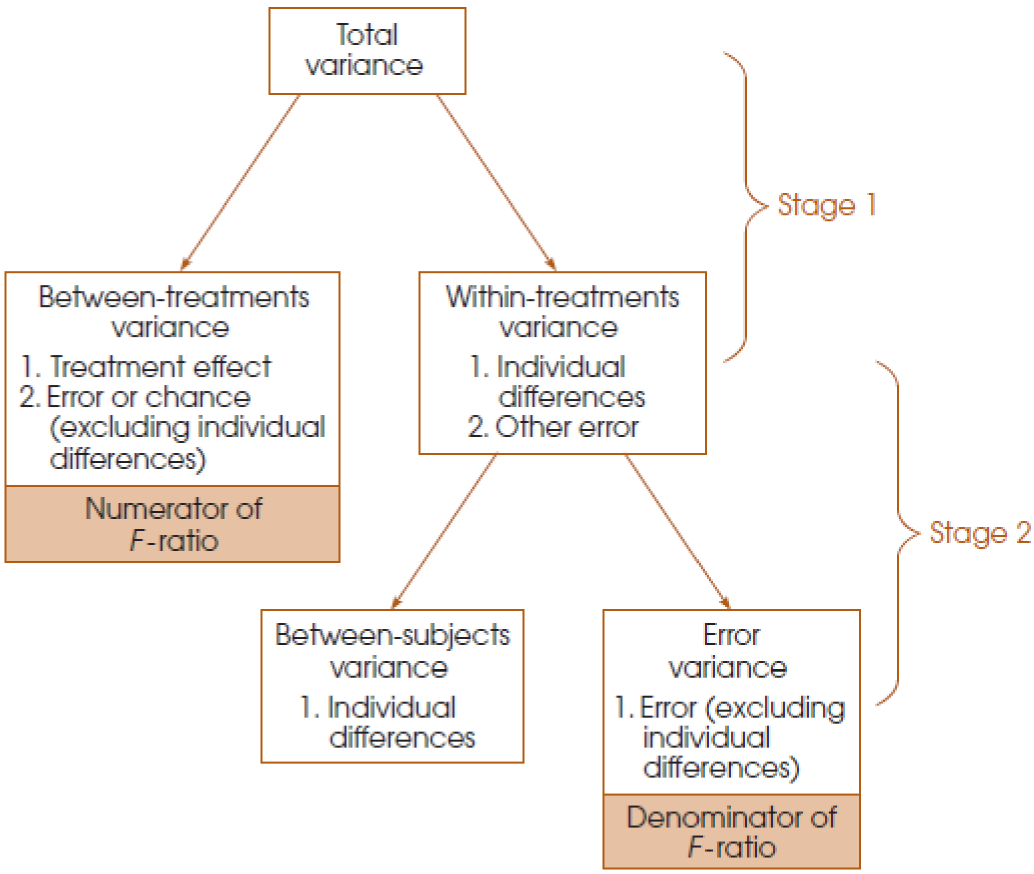


FIGURE 13.2
The partitioning of variance for a repeated-measures experiment.

DEFINITION:In a repeated-measures ANOVA, the denominator of the *F*-ratio is called the **residual variance,** or the **error variance,** and measures how much variance is expected if there are no systematic treatment effects and no individual differences contributing to the variability of the scores.

DEMONSTRATION 13.1

REPEATED-MEASURES ANOVA

The following data were obtained from a research study examining the effect of sleep deprivation on motor-skills performance. A sample of five participants was tested on a motor-skills task after 24 hours of sleep deprivation, tested again after 36 hours, and tested once more after 48 hours. The dependent variable is the number of errors made on the motor-skills task. Do these data indicate that the number of hours of sleep deprivation has a significant effect on motor skills performance?

Participant	24 Hours	36 Hours	48 Hours	P totals	
A	0	0	6	6	$N = 15$
B	1	3	5	9	$G = 45$
C	0	1	5	6	$\Sigma X^2 = 245$
D	4	5	9	18	
E	0	1	5	6	
	$T = 5$ $SS = 12$	$T = 10$ $SS = 16$	$T = 30$ $SS = 12$		

STEP 1 **State the hypotheses, and specify alpha.** The null hypothesis states that, for the general population, there are no differences among the three deprivation conditions. Any differences that exist among the samples are simply the result of chance or error. In symbols,

$$H_0: \mu_1 = \mu_2 = \mu_3$$

The alternative hypothesis states that there are differences among the conditions.

H_1: At least one of the treatment means is different.

$$SS_{\text{total}} = \Sigma X^2 - \frac{G^2}{N} = 245 - \frac{45^2}{15} = 110$$

$$SS_{\text{within}} = \Sigma SS_{\text{inside each treatment}} = 12 + 16 + 12 = 40$$

$$SS_{\text{between}} = \Sigma \frac{T^2}{n} - \frac{G^2}{N} = \frac{5^2}{5} + \frac{10^2}{5} + \frac{30^2}{5} - \frac{45^2}{15} = 70$$

and the corresponding degrees of freedom are

$$df_{\text{total}} = N = -1 = 14$$

$$df_{\text{within}} = \Sigma df = 4 + 4 + 4 = 12$$

$$df_{\text{between}} = k - 1 = 2$$

STAGE 2 The second stage of the repeated-measures analysis measures and removes the individual differences from the denominator of the F-ratio.

$$SS_{\text{between subjects}} = \Sigma \frac{P^2}{k} - \frac{G^2}{N}$$

$$= \frac{6^2}{3} + \frac{9^2}{3} + \frac{6^2}{3} + \frac{18^2}{3} + \frac{6^2}{3} - \frac{45^2}{15}$$

$$= 36$$

$$SS_{\text{error}} = SS_{\text{within}} = SS_{\text{between subjects}}$$

$$= 40 - 36$$

$$= 4$$

and the corresponding df values are

$$df_{\text{between subjects}} = n - 1 = 4$$

$$df_{\text{error}} = df_{\text{within}} - df_{\text{between subjects}}$$

$$= 12 - 4$$

The mean square values that form the F-ratio are as follows:

$$MS_{\text{between}} = \frac{SS_{\text{between}}}{df_{\text{between}}} = \frac{70}{2} = 35$$

$$MS_{\text{error}} = \frac{SS_{\text{error}}}{df_{\text{error}}} = \frac{4}{8} = 0.50$$

Finally, the F-ratio is

$$F = \frac{MS_{\text{between}}}{MS_{\text{error}}} = \frac{35}{0.50} = 70.00$$

STEP 3 **Make a decision and state a conclusion.** With $df = 2, 8$ and $\alpha = .05$, the critical value is $F = 4.46$. Our obtained F-ratio ($F = 70.00$) is well into the critical region, so our decision is to reject the null hypothesis and conclude that there are significant differences among the three levels of sleep deprivation.

TRATION 13.2

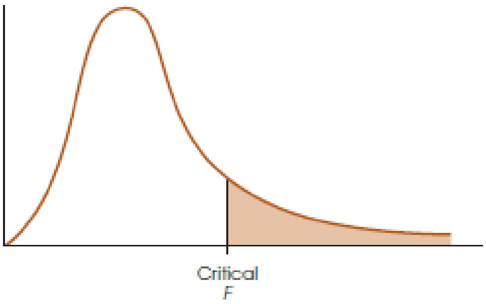
EFFECT SIZE FOR THE REPEATED-MEASURES ANOVA

We compute η^2, the percentage of variance explained by the treatment differences, for the data in Demonstration 13.1. Using Equation 13.11 we obtain

$$\eta^2 = \frac{SS_{\text{between treatments}}}{SS_{\text{between treatments}} + SS_{\text{error}}} = \frac{70}{70 + 4} = \frac{70}{74} = 0.95 \quad \text{(or 95\%)}$$

TABLE B.4 THE *F* DISTRIBUTION*

*Table entries in lightface type are critical values for the .05 level of significance.
Boldface type values are for the .01 level of significance.



Degrees of Freedom: Denominator	Degrees of Freedom: Numerator														
	1	2	3	4	5	6	7	8	9	10	11	12	14	16	20
1	161	200	216	225	230	234	237	239	241	242	243	244	245	246	248
	4052	**4999**	**5403**	**5625**	**5764**	**5859**	**5928**	**5981**	**6022**	**6056**	**6082**	**6106**	**6142**	**6169**	**6208**
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39	19.40	19.41	19.42	19.43	19.44
	98.49	**99.00**	**99.17**	**99.25**	**99.30**	**99.33**	**99.34**	**99.36**	**99.38**	**99.40**	**99.41**	**99.42**	**99.43**	**99.44**	**99.45**
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78	8.76	8.74	8.71	8.69	8.66
	34.12	**30.92**	**29.46**	**28.71**	**28.24**	**27.91**	**27.67**	**27.49**	**27.34**	**27.23**	**27.13**	**27.05**	**26.92**	**26.83**	**26.69**
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.93	5.91	5.87	5.84	5.80
	21.20	**18.00**	**16.69**	**15.98**	**15.52**	**15.21**	**14.98**	**14.80**	**14.66**	**14.54**	**14.45**	**14.37**	**14.24**	**14.15**	**14.02**
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74	4.70	4.68	4.64	4.60	4.56
	16.26	**13.27**	**12.06**	**11.39**	**10.97**	**10.67**	**10.45**	**10.27**	**10.15**	**10.05**	**9.96**	**9.89**	**9.77**	**9.68**	**9.55**
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.96	3.92	3.87
	13.74	**10.92**	**9.78**	**9.15**	**8.75**	**8.47**	**8.26**	**8.10**	**7.98**	**7.87**	**7.79**	**7.72**	**7.60**	**7.52**	**7.39**
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63	3.60	3.57	3.52	3.49	3.44
	12.25	**9.55**	**8.45**	**7.85**	**7.46**	**7.19**	**7.00**	**6.84**	**6.71**	**6.62**	**6.54**	**6.47**	**6.35**	**6.27**	**6.15**
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34	3.31	3.28	3.23	3.20	3.15
	11.26	**8.65**	**7.59**	**7.01**	**6.63**	**6.37**	**6.19**	**6.03**	**5.91**	**5.82**	**5.74**	**5.67**	**5.56**	**5.48**	**5.36**
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13	3.10	3.07	3.02	2.98	2.93
	10.56	**8.02**	**6.99**	**6.42**	**6.06**	**5.80**	**5.62**	**5.47**	**5.35**	**5.26**	**5.18**	**5.11**	**5.00**	**4.92**	**4.80**
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97	2.94	2.91	2.86	2.82	2.77
	10.04	**7.56**	**6.55**	**5.99**	**5.64**	**5.39**	**5.21**	**5.06**	**4.95**	**4.85**	**4.78**	**4.71**	**4.60**	**4.52**	**4.41**
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86	2.82	2.79	2.74	2.70	2.65
	9.65	**7.20**	**6.22**	**5.67**	**5.32**	**5.07**	**4.88**	**4.74**	**4.63**	**4.54**	**4.46**	**4.40**	**4.29**	**4.21**	**4.10**
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76	2.72	2.69	2.64	2.60	2.54
	9.33	**6.93**	**5.95**	**5.41**	**5.06**	**4.82**	**4.65**	**4.50**	**4.39**	**4.30**	**4.22**	**4.16**	**4.05**	**3.98**	**3.86**
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67	2.63	2.60	2.55	2.51	2.46
	9.07	**6.70**	**5.74**	**5.20**	**4.86**	**4.62**	**4.44**	**4.30**	**4.19**	**4.10**	**4.02**	**3.96**	**3.85**	**3.78**	**3.67**
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70	2.65	2.60	2.56	2.53	2.48	2.44	2.39
	8.86	**6.51**	**5.56**	**5.03**	**4.69**	**4.46**	**4.28**	**4.14**	**4.03**	**3.94**	**3.86**	**3.80**	**3.70**	**3.62**	**3.51**
15	4.54	3.68	3.29	3.06	2.90	2.79	2.70	2.64	2.59	2.55	2.51	2.48	2.43	2.39	2.33
	8.68	**6.36**	**5.42**	**4.89**	**4.56**	**4.32**	**4.14**	**4.00**	**3.89**	**3.80**	**3.73**	**3.67**	**3.56**	**3.48**	**3.36**
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.45	2.42	2.37	2.33	2.28
	8.53	**6.23**	**5.29**	**4.77**	**4.44**	**4.20**	**4.03**	**3.89**	**3.78**	**3.69**	**3.61**	**3.55**	**3.45**	**3.37**	**3.25**

IN THE LITERATURE

REPORTING THE RESULTS OF A REPEATED-MEASURES ANOVA

As described in Chapter 12 (p. 409), the format for reporting ANOVA results in journal articles consists of

1. A summary of descriptive statistics (at least treatment means and standard deviations, and tables or graphs as needed)
2. A concise statement of the outcome of the ANOVA

For the study in Example 13.1, the report could state:

> The means and variances for the four television viewing distances are shown in Table 1. A repeated-measures analysis of variance indicated significant mean differences in the participants' ratings of the four distances, $F(3, 12) = 24.88$, $p < .01$, $\eta^2 = 0.862$.
>
> **TABLE 1**
>
> Ratings of satisfaction with different television-viewing distances

	9 Feet	12 Feet	15 Feet	18 Feet
M	1.00	2.00	5.00	4.00
SD	1.41	1.41	1.58	1.22

SPSS OUTPUT

Descriptive Statistics

	Mean	Std. Deviation	N
VAR00001	1.0000	1.41421	5
VAR00002	2.0000	1.41421	5
VAR00003	5.0000	1.58114	5
VAR00004	4.0000	1.22474	5

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
factor 1	Sphericity Assumed	50.000	3	16.667	25.000	.000
	Greenhouse-Geisser	50.000	1.600	31.250	25.000	.001
	Huynh-Feldt	50.000	2.500	20.000	25.000	.000
	Lower-bound	50.000	1.000	50.000	25.000	.007
Error (factor 1)	Sphericity Assumed	8.000	12	.667		
	Greenhouse-Geisser	8.000	6.400	1.250		
	Huynh-Feldt	8.000	10.000	.800		
	Lower-bound	8.000	4.000	2.000		

15. A researcher is evaluating customer satisfaction with the service and coverage of two phone carriers. Each individual in a sample of $n = 25$ uses one carrier for two weeks and then switches to the other. Each participant then rates the two carriers. The following table presents the results from the repeated-measures ANOVA comparing the average ratings. Fill in the missing values in the table. (*Hint:* Start with the *df* values.)

Source	SS	df	MS	
Between treatments	____	____	2	$F =$ ____
Within treatments	____	____		
Between subjects	____	____		
Error	12	____	____	
Total	23	____		

16. The following summary table presents the results from a repeated-measures ANOVA comparing three treatment conditions with a sample of $n = 11$ subjects. Fill in the missing values in the table. (*Hint:* Start with the *df* values.)

Source	SS	df	MS	
Between treatments	____	____	____	$F = 5.00$
Within treatments	80	____		
Between subjects	____	____		
Error	60	____	____	
Total	____	____		

17. The following summary table presents the results from a repeated-measures ANOVA comparing four treatment conditions, each with a sample of $n = 12$ participants. Fill in the missing values in the table. (*Hint:* Start with the *df* values.)

Source	SS	df	MS	
Between treatments	54	____	20	$F =$ ____
Within treatments	____	____		
Between subjects	____	____		
Error	____	____	3	
Total	194	____		

1. How does the denominator of the F-ratio (the error term) differ for a repeated-measures ANOVA compared to an independent-measures ANOVA?

2. The repeated-measures ANOVA can be viewed as a two-stage process. What is the purpose of the second stage?

3. A researcher conducts an experiment comparing three treatment conditions with $n = 10$ scores in each condition.
   a. If the researcher uses an independent-measures design, how many individuals are needed for the study and what are the *df* values for the F-ratio?
   b. If the researcher uses a repeated-measures design, how many individuals are needed for the study and what are the *df* values for the F-ratio?

4. A researcher conducts a repeated-measures experiment using a sample of $n = 8$ subjects to evaluate the differences among four treatment conditions. If the results are examined with an ANOVA, what are the *df* values for the F-ratio?

5. A researcher uses a repeated-measures ANOVA to evaluate the results from a research study and reports an F-ratio with $df = 2, 30$.
   a. How many treatment conditions were compared in the study?
   b. How many individuals participated in the study?

6. A published report of a repeated-measures research study includes the following description of the statistical analysis. "The results show significant differences among the treatment conditions, $F(2, 20) = 6.10$, $p < .01$."
   a. How many treatment conditions were compared in the study?
   b. How many individuals participated in the study?