1. For a normal distribution with a mean of $\mu = 60$ and a standard deviation of $\sigma = 12$, find each probability value requested.
   a. $p(X > 66)$
   b. $p(X < 75)$
   c. $p(X < 57)$
   d. $p(48 < X < 72)$

2. Scores on the Mathematics section of the SAT Reasoning Test form a normal distribution with a mean of $\mu = 500$ and a standard deviation of $\sigma = 100$.
   a. If the state college only accepts students who score in the top 60% on this test, what is the minimum score needed for admission?
   b. What is the minimum score necessary to be in the top 10% of the distribution?
   c. What scores form the boundaries for the middle 50% of the distribution?

3. What is the probability of selecting a score greater than 45 from a positively skewed distribution with $\mu = 40$ and $\sigma = 10$? (Be careful.)

**ANSWERS**

1. a. $p = 0.3085$
   b. $p = 0.8944$
   c. $p = 0.4013$
   d. $p = 0.6826$

2. a. $z = -0.25; X = 475$
   b. $z = 1.28; X = 628$
   c. $z = \pm 0.67; X \approx 433$ and $X \approx 567$

3. You cannot obtain the answer. The unit normal table cannot be used to answer this question because the distribution is not normal.

1. Under what circumstances is the normal distribution an accurate approximation of the binomial distribution?

2. In the game Rock-Paper-Scissors, the probability that both players will select the same response and tie is $p = \frac{1}{3}$, and the probability that they will pick different responses is $p = \frac{2}{3}$. If two people play 72 rounds of the game and choose their responses randomly, what is the probability that they will choose the same response (tie) more than 28 times?

3. If you toss a balanced coin 36 times, you would expect, on the average, to get 18 heads and 18 tails. What is the probability of obtaining exactly 18 heads in 36 tosses?

**ANSWERS**

1. When $p n$ and $q n$ are both greater than 10

2. With $p = \frac{1}{3}$ and $q = \frac{2}{3}$, the binomial distribution is normal with $\mu = 24$ and $\sigma = 4$;
   
   $p(X > 28.5) = p(z > 1.13) = 0.1292$.

3. $X = 18$ is an interval with real limits of 17.5 and 18.5. The real limits correspond to $z = \pm 0.17$, and a probability of $p = 0.1350$. 
### SOME SELECTED EXERCISES

13. A normal distribution has a mean of $\mu = 50$ and a standard deviation of $\sigma = 12$. For each of the following scores, indicate whether the tail is to the right or left of the score and find the proportion of the distribution located in the tail.
   a. $X = 53$
   b. $X = 44$
   c. $X = 68$
   d. $X = 38$

15. The distribution of scores on the SAT is approximately normal with a mean of $\mu = 500$ and a standard deviation of $\sigma = 100$. For the population of students who have taken the SAT,
   a. What proportion have SAT scores greater than 700?
   b. What proportion have SAT scores greater than 550?
   c. What is the minimum SAT score needed to be in the highest 10% of the population?
   d. If the state college only accepts students from the top 60% of the SAT distribution, what is the minimum SAT score needed to be accepted?

16. The distribution of SAT scores is normal with $\mu = 500$ and $\sigma = 100$.
   a. What SAT score, $X$ value, separates the top 15% of the distribution from the rest?
   b. What SAT score, $X$ value, separates the top 10% of the distribution from the rest?
   c. What SAT score, $X$ value, separates the top 2% of the distribution from the rest?

18. Information from the Department of Motor Vehicles indicates that the average age of licensed drivers is $\mu = 45.7$ years with a standard deviation of $\sigma = 12.5$ years. Assuming that the distribution of drivers’ ages is approximately normal,
   a. What proportion of licensed drivers are older than 50 years old?
   b. What proportion of licensed drivers are younger than 30 years old?

22. A multiple-choice test has 48 questions, each with four response choices. If a student is simply guessing at the answers,
   a. What is the probability of guessing correctly for any one question?
   b. On average, how many questions would a student get correct for the entire test?
   c. What is the probability that a student would get more than 15 answers correct simply by guessing?
   d. What is the probability that a student would get 15 or more answers correct simply by guessing?

23. A true/false test has 40 questions. If a student is simply guessing at the answers,
   a. What is the probability of guessing correctly for any one question?
   b. On average, how many questions would the student get correct for the entire test?
   c. What is the probability that the student would get more than 25 answers correct simply by guessing?
   d. What is the probability that the student would get 25 or more answers correct simply by guessing?

### Additional Exercises

10. Find the $z$-score location of a vertical line that separates a normal distribution as described in each of the following.
   a. 20% in the tail on the left
   b. 40% in the tail on the right
   c. 75% in the body on the left
   d. 99% in the body on the right

11. Find the $z$-score boundaries that separate a normal distribution as described in each of the following.
   a. The middle 20% from the 80% in the tails.
   b. The middle 50% from the 50% in the tails.
   c. The middle 95% from the 5% in the tails.
   d. The middle 99% from the 1% in the tails.

19. A consumer survey indicates that the average household spends $\mu = 185$ on groceries each week. The distribution of spending amounts is approximately normal with a standard deviation of $\sigma = 25$. Based on this distribution,
   a. What proportion of the population spends more than $200 per week on groceries?
   b. What is the probability of randomly selecting a family that spends less than $150 per week on groceries?
   c. How much money do you need to spend on groceries each week to be in the top 20% of the distribution?

24. A roulette wheel has alternating red and black numbered slots into one of which the ball finally stops to determine the winner. If a gambler always bets on black to win, what is the probability of winning at least 24 times in a series of 36 spins? (Note that at least 24 wins means 24 or more.)
EXAMPLE WITH SOLUTION

A multiple-choice quiz has 200 questions, each with 4 possible answers of which only 1 is correct. What is the probability that sheer guesswork yields from 25 to 30 correct answers for the 80 of the 200 problems about which the student has no knowledge?

The probability of guessing a correct answer for each of the 80 questions is \( p = 1/4 \). If \( X \) represents the number of correct answers resulting from guesswork, then

\[
P(25 \leq X \leq 30) = \sum_{x=25}^{30} b(x; 80, 1/4).\]

Using the normal curve approximation with

\[
\mu = np = (80) \left( \frac{1}{4} \right) = 20
\]

and

\[
\sigma = \sqrt{npq} = \sqrt{(80)(1/4)(3/4)} = 3.873,
\]

we need the area between \( x_1 = 24.5 \) and \( x_2 = 30.5 \). The corresponding \( z \) values are

\[
z_1 = \frac{24.5 - 20}{3.873} = 1.16 \text{ and } z_2 = \frac{30.5 - 20}{3.873} = 2.71.
\]

The probability of correctly guessing from 25 to 30 questions is given by the shaded region in Figure 6.27. From Table A.3 we find that

\[
P(25 \leq X \leq 30) \approx P(1.16 < Z < 2.71)
\]

\[
= P(Z < 2.71) - P(Z < 1.16) = 0.9966 - 0.8770 = 0.1196. \quad \blacksquare
\]

6.27 The probability that a patient recovers from a delicate heart operation is 0.9. Of the next 100 patients having this operation, what is the probability that
(a) between 84 and 95 inclusive survive?
(b) fewer than 86 survive?

6.33 Statistics released by the National Highway Traffic Safety Administration and the National Safety Council show that on an average weekend night, 1 out of every 10 drivers on the road is drunk. If 400 drivers are randomly checked next Saturday night, what is the probability that the number of drunk drivers will be
(a) less than 32?
(b) more than 49?
(c) at least 35 but less than 47?