Correlation

Correlation is a statistical technique that is used to measure and describe the relationship between two variables. Usually the two variables are simply observed as they exist naturally in the environment—there is no attempt to control or manipulate the variables.

The Characteristics of a Relationship

A correlation is a numerical value that describes and measures three characteristics of the relationship between *X* and *Y*. These three characteristics are as follows:

1. The Direction of the Relationship. The sign of the correlation, positive or negative, describes the direction of the relationship.

Definitions:

In a **positive correlation,** the two variables tend to change in the same direction: As the value of the *X* variable increases from one individual to another, the *Y* variable also tends to increase; when the *X* variable decreases, the *Y* variable also decreases.

In a **negative correlation,** the two variables tend to go in opposite directions. As the *X* variable increases, the *Y* variable decreases. That is, it is an inverse relationship.

2. The Form of the Relationship. In the preceding coffee and beer examples, the relationships tend to have a linear form; that is, the points in the scatter plot tend to cluster around a straight line. We have drawn a line through the middle of the data points in each figure to help show the relationship. The most common use of correlation is to measure straight-line relationships. However, other forms of relationships do exist and there are special correlations used to measure them.

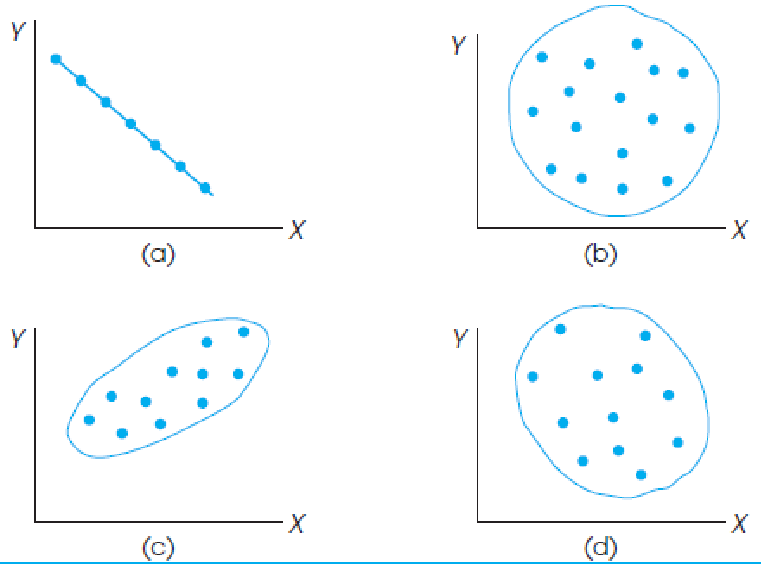
3. The Strength or Consistency of the Relationship. Finally, the correlation measures the consistency of the relationship. For a linear relationship, for example, the data points could fit perfectly on a straight line. Every time *X* increases by one point, the value of *Y* also changes by a consistent and predictable amount. Figure 14.3(a) shows an example of a perfect linear relationship. However, relationships are usually not perfect. Although there may be a tendency for the value of *Y* to increase whenever *X* increases, the amount that *Y* changes is not always the same, and occasionally, *Y* decreases when *X* increases. In this situation, the data points do not fall perfectly on a straight line. The consistency of the relationship is measured by the numerical value of the correlation. A *perfect correlation* always is identified by a correlation of 1.00 and indicates a perfectly consistent relationship. For a correlation of 1.00 (or –1.00), each change in *X* is accompanied by a perfectly predictable change in *Y*. At the other extreme, a correlation of 0 indicates no consistency at all. For a correlation of 0, the data points are scattered randomly with no clear trend [see Figure 14.3(b)]. Intermediate values between 0 and 1 indicate the degree of consistency.

There are three different kinds of correlation that will be discussed in the following order

1- Pearson's r
2- Spearman's ρ
3- Pearson's ϕ
4- The Point-Biserial Correlation

FIGURE 14.3

Examples of different values for linear correlations: (a) a perfect negative correlation, –1.00; (b) no linear trend, 0.00; (c) a strong positive relationship, approximately +0.90; (d) a relatively weak negative correlation, approximately –0.40.



1. The Pearson Correlation

By far the most common correlation is the *Pearson correlation* (or the Pearson product–moment correlation) which measures the degree of straight-line relationship.

Definition

The **Pearson correlation** measures the degree and the direction of the linearrelationship between two variables. The Pearson correlation is identified by the letter *r*. Conceptually, this correlation is computed by

$$r = \frac{\text{degree to which } X \text{ and } Y \text{ vary together}}{\text{degree to which } X \text{ and } Y \text{ vary separately}}$$

$$= \frac{\text{covariability of } X \text{ and } Y}{\text{variability of } X \text{ and } Y \text{ separately}}$$

$$r = \frac{SP}{\sqrt{SS_X SS_Y}}$$

When there is a perfect linear relationship, every change in the *X* variable is accompanied by a corresponding change in the *Y* variable.

EXAMPLE:

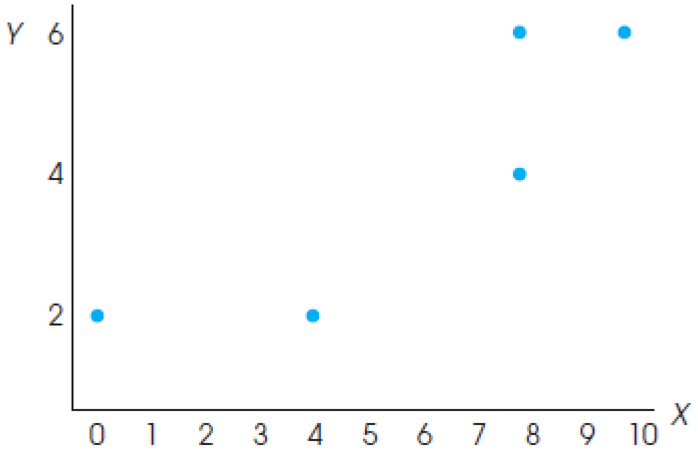
The Pearson correlation is computed for the following set of $n = 5$ pairs of scores.

X	Y
0	2
10	6
4	2
8	4
8	6

Before starting any calculations, it is useful to put the data in a scatter plot and make a preliminary estimate of the correlation. These data have been graphed in Figure 14.4. Looking at the scatter plot, it appears that there is a very good (but not perfect) positive correlation. You should expect an approximate value of $r = +0.8$ or $+0.9$. To find the Pearson correlation, we need SP, SS for X, and SS for Y. The calculations for each of these values, using the definitional formulas, are presented in Table 14.1. (Note that the mean for the X values is $M_X = 6$ and the mean for the Y scores is $M_Y = 4$.)

Using the values from Table 14.1, the Pearson correlation is

$$r = \frac{SP}{\sqrt{(SS_X)(SS_Y)}} = \frac{28}{\sqrt{(64)(16)}} = \frac{28}{32} = +0.875$$



Scores		Deviations		Squared Deviations		Products
X	Y	$X - M_X$	$Y - M_Y$	$(X - M_X)^2$	$(Y - M_Y)^2$	$(X - M_X)(Y - M_Y)$
0	2	−6	−2	36	4	+12
10	6	+4	+2	16	4	+8
4	2	−2	−2	4	4	+4
8	4	+2	0	4	0	0
8	6	+2	+2	4	4	+4
				$SS_X = 64$	$SS_Y = 16$	$SP = +28$

The Pearson Correlation and *z*-Scores

The Pearson correlation measures the relationship between an individual's location in the *X* distribution and his or her location in the *Y* distribution. For example, a positive correlation means that individuals who have a high *X* score also tend to have a high *Y* score. Similarly, a negative correlation indicates that individuals with high *X* scores tend to have low *Y* scores.

z-scores identify the exact location of each individual score within a distribution. With this in mind, each *X* value can be transformed into a *z*-score, z_X, using the mean and standard deviation for the set of *X*s. Similarly, each *Y* score can be transformed into z_Y. If the *X* and *Y* values are viewed as a sample, then the transformation is completed using the sample formula for *z* . If the *X* and *Y* values form a **complete population**, then the *z*-scores are computed . After the transformation, the formula for the Pearson correlation can be expressed entirely in terms of *z*-scores

$$\text{For a sample, } r = \frac{\Sigma z_X z_y}{(n-1)} \qquad \text{For a population, } \rho = \frac{\Sigma z_X z_y}{N}$$

Using and Interpreting the Pearson Correlation

Where and Why Correlations Are Used

1. Prediction. If two variables are known to be related in a systematic way, then it is possible to use one of the variables to make predictions about the other.

2. Validity. Suppose that a psychologist develops a new test for measuring intelligence. How could you show that this test truly measures what it claims; that is, how could you demonstrate the validity of the test? One common technique for demonstrating validity is to use a correlation. If the test actually measures intelligence, then the scores on the test should be related to other measures of intelligence—for example, standardized IQ tests, performance on learning tasks, problem-solving ability, and so on. The psychologist could measure the correlation between the new test and each of these other measures of intelligence to demonstrate that the new test is valid.

3. Reliability. In addition to evaluating the validity of a measurement procedure, correlations are used to determine reliability. A measurement procedure is considered reliable to the extent that it produces stable, consistent measurements. That is, a reliable measurement procedure produces the same (or nearly the same) scores when the same individuals are measured twice under the same conditions. For example, if your IQ were measured as 113 last week, you would expect to obtain nearly the same score if your IQ were measured again this week. One way to evaluate reliability is to use correlations to determine the relationship between two sets of measurements. When reliability is high, the correlation between two measurements should be strong and positive.

4. Theory Verification. Many psychological theories make specific predictions about the relationship between two variables. For example, a theory may predict a relationship between brain size and learning ability; a developmental theory may predict a relationship between the parents' IQs and the child's IQ; a social psychologist may have a theory predicting a relationship between personality type and behavior in a social situation. In each case, the prediction of the theory could be tested by determining the correlation between the two variables.

Interpreting Correlations

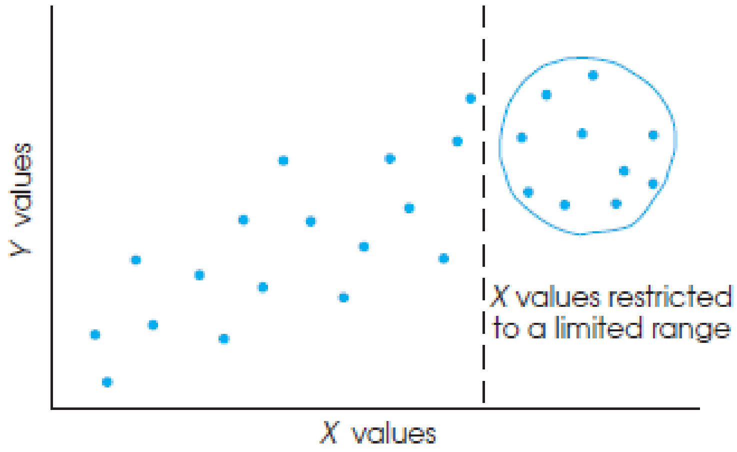
When you encounter correlations, there are four additional considerations that you should bear in mind:

1. Correlation simply describes a relationship between two variables. It does not explain why the two variables are related. Specifically, a correlation should not and cannot be interpreted as proof of a cause-and-effect relationship between the two variables.

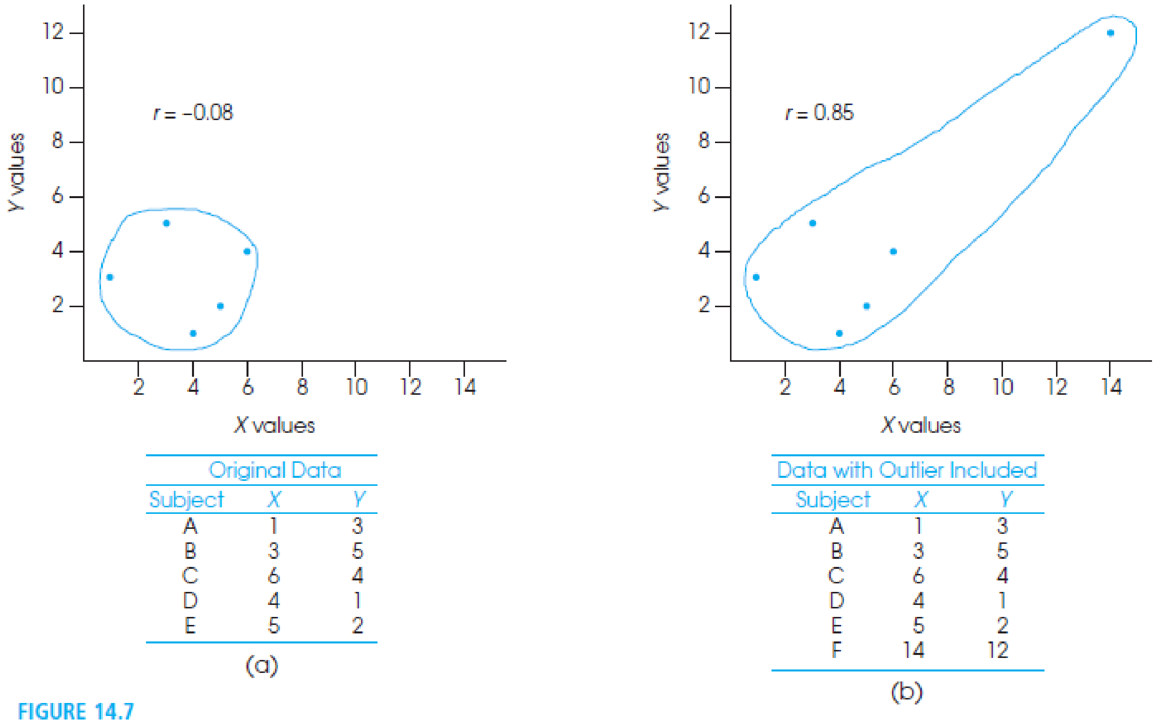
2. The value of a correlation can be affected greatly by the range of scores represented in the data.

FIGURE 14.6

In this example, the full range of X and Y values shows a strong, positive correlation, but the restricted range of scores produces a correlation near zero.



3. One or two extreme data points, often called *outliers*, can have a dramatic effect on the value of a correlation.



Original Data		
Subject	X	Y
A	1	3
B	3	5
C	6	4
D	4	1
E	5	2

Data with Outlier Included		
Subject	X	Y
A	1	3
B	3	5
C	6	4
D	4	1
E	5	2
F	14	12

FIGURE 14.7

A demonstration of how one extreme data point (an outlier) can influence the value of a correlation.

4. When judging how “good” a relationship is, it is tempting to focus on the numerical value of the correlation. For example, a correlation of +0.5 is halfway between 0 and 1.00 and, therefore, appears to represent a moderate degree of relationship. However, a correlation should not be interpreted as a proportion. Although a correlation of 1.00 does mean that there is a 100% perfectly predictable relationship between X and Y, a correlation of 0.5 does not mean that you can make predictions with 50% accuracy. To describe how accurately one variable predicts the other, you must square the correlation. Thus, a correlation of r = 0.5 means that one variable *partially* predicts the other, but the predictable portion is only $r^2 = 0.5^2 = 0.25$ (or 25%) of the total variability.

Definition:

The value r^2 is called the **coefficient of determination** because it measures the proportion of variability in one variable that can be determined from the relationship with the other variable. A correlation of $r = 0.80$ (or –0.80), for example, means that $r^2 = 0.64$ (or 64%) of the variability in the *Y* scores can be predicted from the relationship with *X*.

We introduced r^2 as a method for measuring effect size for research studies where mean differences were used to compare treatments. Specifically, we measured how much of the variance in the scores was accounted for by the differences between treatments. In experimental terminology, r^2 measures how much of the variance in the dependent variable is accounted for by the independent variable. Now we are doing the same thing, except that there is no independent or dependent variable. Instead, we simply have two variables, *X* and *Y*, and we use r^2 to measure how much of the variance in one variable can be determined from its relationship with the other variable.

Hypothesis Tests with the Pearson Correlation

The hypothesis test evaluating the significance of a correlation can be conducted using either a *t* statistic or an *F*-ratio. The *F*-ratio and we focus on the *t* statistic here. The *t* statistic for a correlation has the same general structure as *t* statistics

$$t = \frac{sample\ statistics - population\ parameter}{standard\ error}$$

In this case, the sample statistic is the sample correlation (*r*) and the corresponding parameter is the population correlation (ρ). The null hypothesis specifies that the population correlation is ρ=0 . The final part of the equation is the standard error, which is determined by

$$standard\ error\ for\ r = s_r = \sqrt{\frac{1 - r^2}{n - 2}}$$

Thus, the complete *t* statistic is

$$t = \frac{r - \rho}{\sqrt{\frac{1 - r^2}{n - 2}}} = \frac{r}{s_r}$$

The *t* statistic has degrees of freedom defined by $df = n – 2$. An intuitive explanation for this value is that a sample with only $n = 2$ data points has no degrees of freedom. Specifically, if there are only two points, they will fit perfectly on a straight line, and the sample produces a perfect correlation of $r =+1.00$ or $r = –1.00$. Because the first two points always produce a perfect correlation, the sample correlation is free to vary only when the data set contains more than two points. Thus, $df = n – 2$.

Example

A researcher is using a regular, two-tailed test with a α= .05 to determine whether a nonzero correlation exists in the population. A sample of *n* =30 individuals is obtained and produces a correlation of *r* = 0.35. The null hypothesis states that there is no correlation in the population.

H_0: ρ=0

For this example, *df* = 28 and the critical values are *t* = ±2.048. With $r^2 = 0.35^2 = 0.1225$, the data produce

$$t = \frac{0.35 - 0}{\sqrt{\frac{1 - 0.1225}{28}}} = \frac{0.35}{0.177} = 1.97$$

The *t* value is not in the critical region so we fail to reject the null hypothesis. The sample correlation is not large enough to reject the null hypothesis.

Partial Correlation

Occasionally a researcher may suspect that the relationship between two variables is being distorted by the influence of a third variable. A **partial correlation** measures the relationship between two variables while controlling the influence of a third variable by holding it constant.

Example

The data points for the 15 cities are shown in the scatter plot in Figure 14.10. Notice that the population variable, *Z*, separates the scores into three distinct groups: When *Z* = 1, the population is low and churches and crime (*X* and *Y*) are also low; when *Z* = 2, the population is moderate and churches and crime (*X* and *Y*) are also moderate; and when *Z* = 3, the population is large and churches and crime are both high. Thus, as the population increases from one city to another, the number of churches and crimes also increase, and the result is a strong positive correlation between churches and crime. Within each of the three population categories, however, there is no linear relationship between churches and crime. Specifically, within each group, the population variable is constant and the five data points for *X* and *Y* form a circular pattern, indicating no consistent linear relationship. The partial correlation allows us to hold population constant across the entire sample and measure the underlying relationship between churches and crime without any influence from population

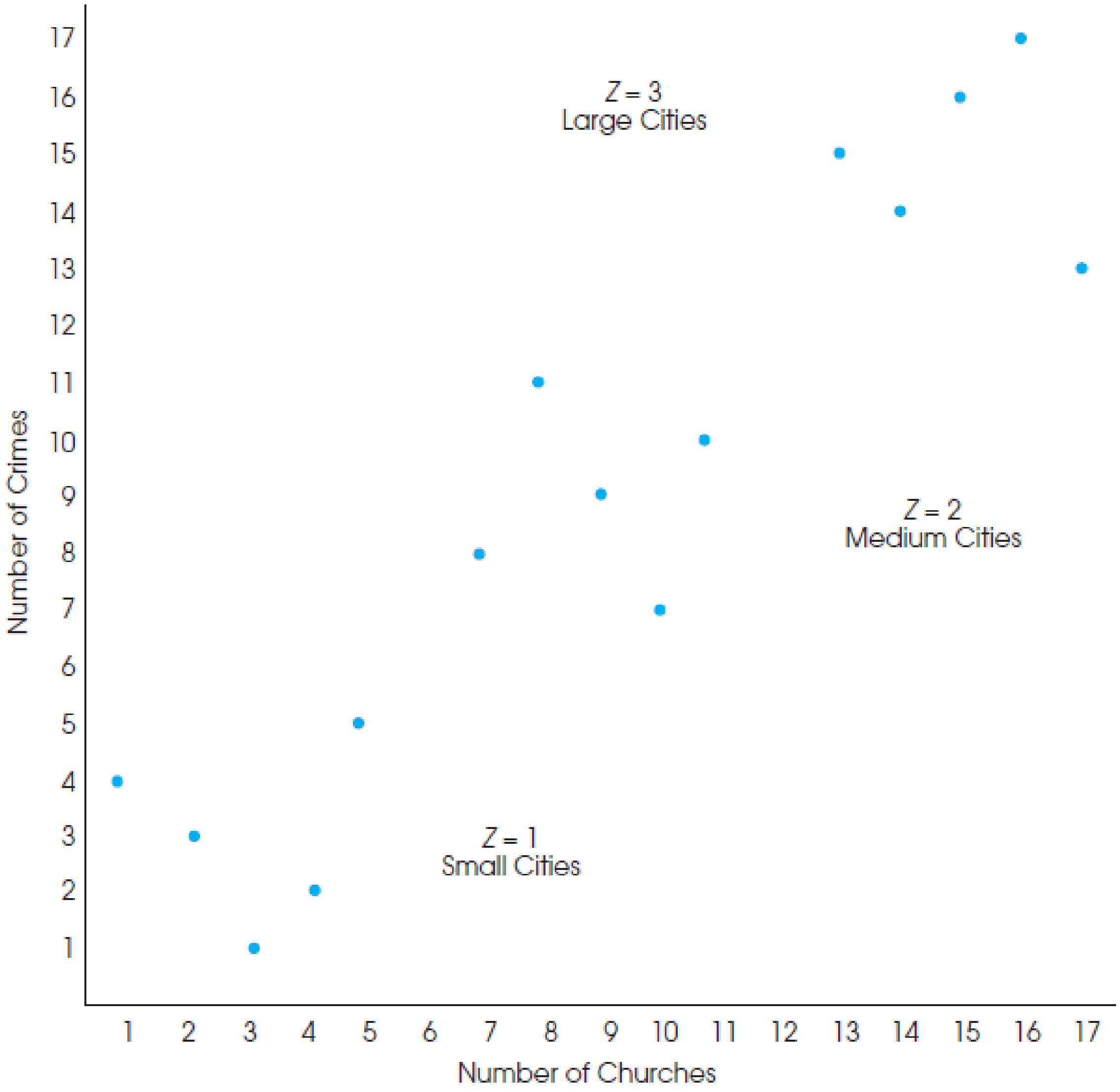


FIGURE 14.10

Hypothetical data showing the relationship between the number of churches and the number of crimes for three groups of cities. Those with small populations ($Z = 1$), those with medium populations ($Z = 2$), and those with large populations ($Z = 3$).

2. The Spearman ρ Correlation

When the Pearson correlation formula is used with data from an ordinal scale (ranks), the result is called the *Spearman correlation*. The Spearman correlation is used in two situations.

First, the Spearman correlation is used to measure the relationship between *X* and *Y* when both variables are measured on ordinal scales. Recall an ordinal scale typically involves ranking individuals rather than obtaining numerical scores. Rank order data are fairly common because they are often easier to obtain than interval or ratio scale data.

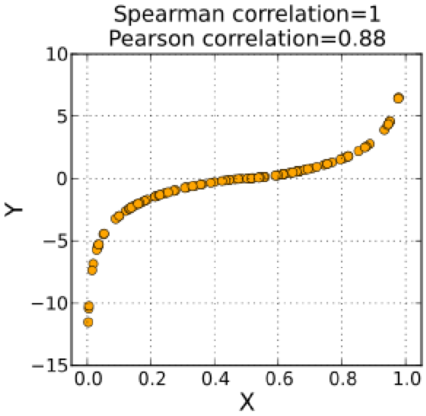
In addition to measuring relationships for ordinal data, the Spearman correlation can be used as a valuable alternative to the Pearson correlation, even when the original raw scores are on an interval or a ratio scale. As we have noted, the Pearson correlation measures the degree of *linear relationship* between two variables—that is, how well the data points fit on a straight line. However, a researcher often expects the data to show a consistently one-directional relationship but not necessarily a linear relationship.

To summarize, the Spearman correlation measures the relationship between two variables when both are measured on ordinal scales (ranks). There are two general situations in which the Spearman correlation is used:

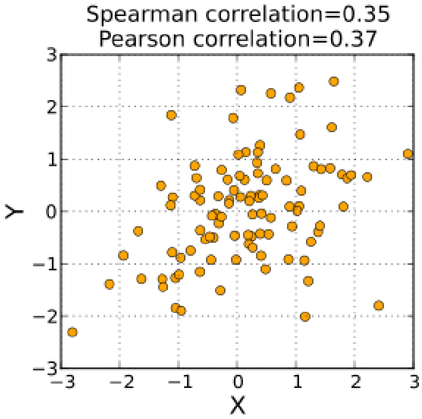
1. Spearman is used when the original data are ordinal. In this case, you rank the ordinal scores and apply the Pearson correlation formula to the set of ranks.

2. Spearman is used when the original scores are numerical values from an interval or ratio scale and the goal is to measure the consistency of the relationship between *X* and *Y*, independent of the specific form of the relationship. In this case, the original scores are first converted to ranks, and then the Pearson correlation formula is used with the ranks. Because the Pearson formula measures the degree to which the ranks fit on a straight line, it also measures the degree of consistency in the relationship for the original scores. Incidentally, when there is a consistently one-directional relationship between two variables, the relationship is said to be *monotonic*. Thus, the Spearman correlation measures the degree of monotonic relationship between two variables.

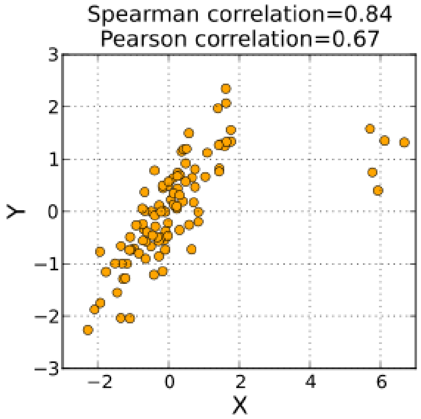
NOTE:The word *monotonic* describes a sequence that is consistently increasing (or decreasing). Like the word *monotonous,* it means constant and unchanging.



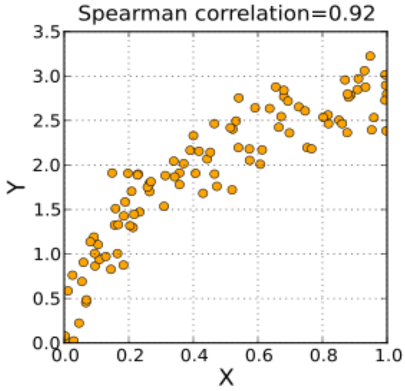
A Spearman correlation of 1 results when the two variables being compared are monotonically related, even if their relationship is not linear. This means that all data-points with greater x-values than that of a given data-point will have greater y-values as well. In contrast, this does not give a perfect Pearson correlation.



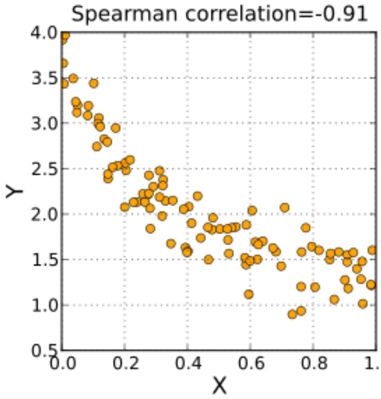
When the data are roughly elliptically distributed and there are no prominent outliers, the Spearman correlation and Pearson correlation give similar values.



The Spearman correlation is less sensitive than the Pearson correlation to strong outliers that are in the tails of both samples. That is because Spearman's rho limits the outlier to the value of its rank.



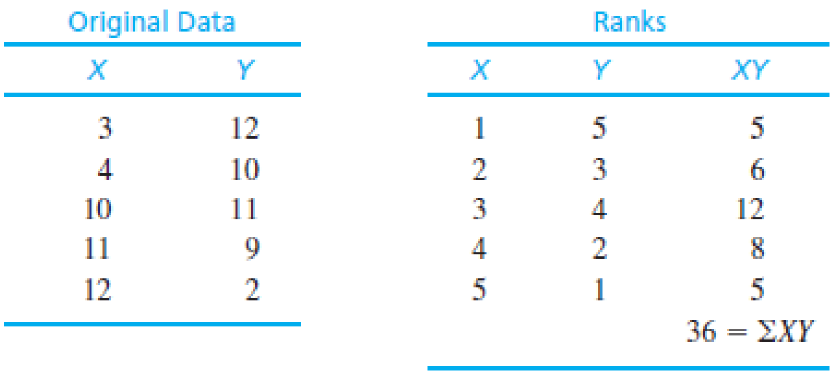
A positive Spearman correlation coefficient corresponds to an increasing monotonic trend between *X* and *Y*.



A negative Spearman correlation coefficient corresponds to a decreasing monotonic trend between*X* and *Y*.

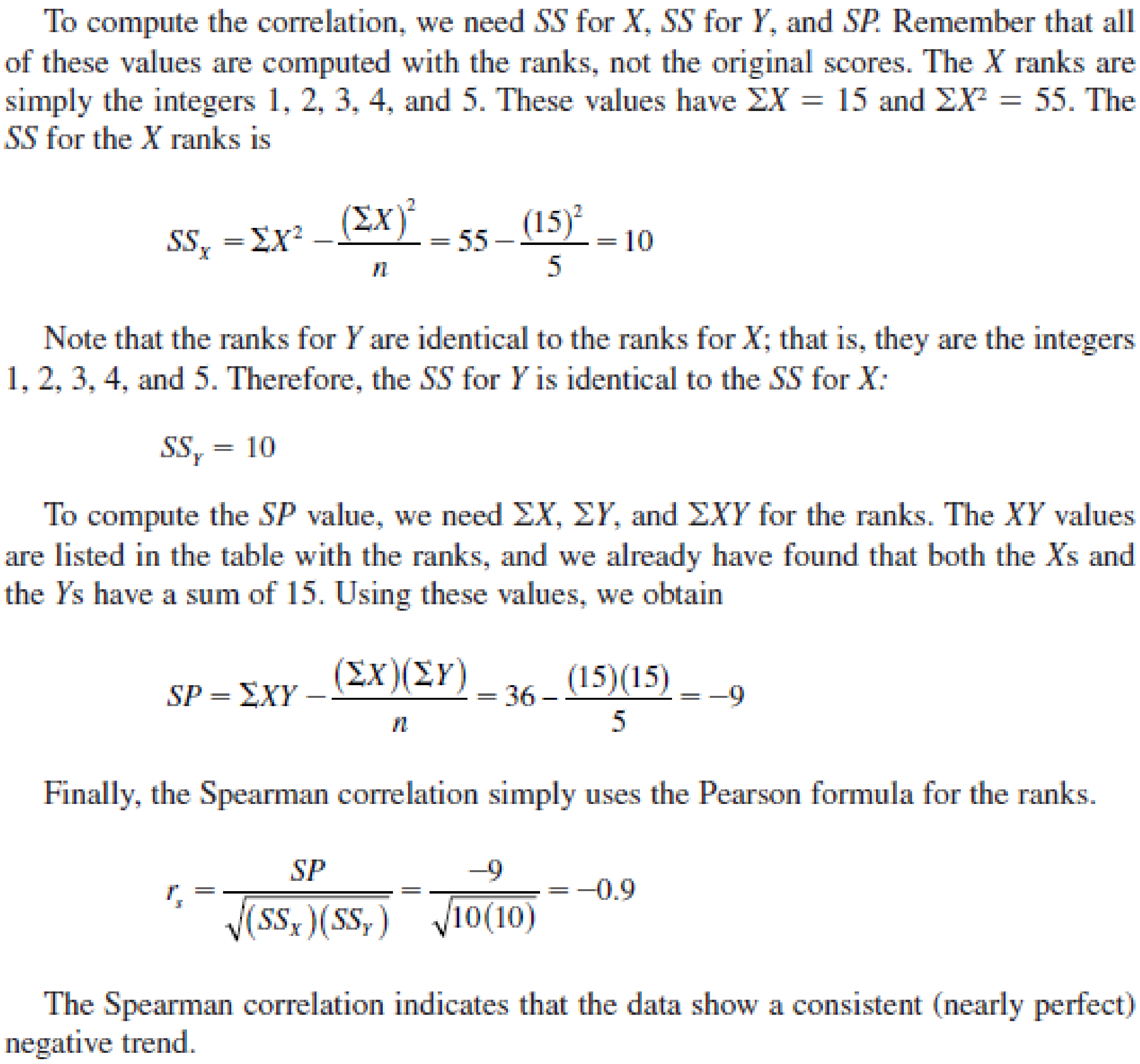
Example

The following data show a nearly perfect monotonic relationship between *X* and *Y*. When *X* increases, *Y* tends to decrease, and there is only one reversal in this general trend. To compute the Spearman correlation, we first rank the *X* and *Y* values, and we then compute the Pearson correlation for the ranks.



Original Data	
X	*Y*
3	12
4	10
10	11
11	9
12	2

Ranks		
X	*Y*	*XY*
1	5	5
2	3	6
3	4	12
4	2	8
5	1	5
		36 = Σ*XY*



To compute the correlation, we need *SS* for *X*, *SS* for *Y*, and *SP*. Remember that all of these values are computed with the ranks, not the original scores. The *X* ranks are simply the integers 1, 2, 3, 4, and 5. These values have $\Sigma X = 15$ and $\Sigma X^2 = 55$. The *SS* for the *X* ranks is

$$SS_X = \Sigma X^2 - \frac{(\Sigma X)^2}{n} = 55 - \frac{(15)^2}{5} = 10$$

Note that the ranks for *Y* are identical to the ranks for *X*; that is, they are the integers 1, 2, 3, 4, and 5. Therefore, the *SS* for *Y* is identical to the *SS* for *X:*

$$SS_Y = 10$$

To compute the *SP* value, we need Σ*X*, Σ*Y*, and Σ*XY* for the ranks. The *XY* values are listed in the table with the ranks, and we already have found that both the *X*s and the *Y*s have a sum of 15. Using these values, we obtain

$$SP = \Sigma XY - \frac{(\Sigma X)(\Sigma Y)}{n} = 36 - \frac{(15)(15)}{5} = -9$$

Finally, the Spearman correlation simply uses the Pearson formula for the ranks.

$$r_s = \frac{SP}{\sqrt{(SS_X)(SS_Y)}} = \frac{-9}{\sqrt{10(10)}} = -0.9$$

The Spearman correlation indicates that the data show a consistent (nearly perfect) negative trend.

3. Pearson’s ϕ correlation

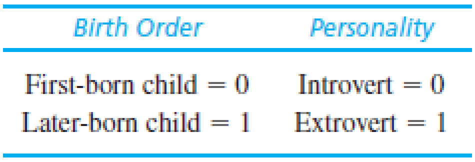
When both variables (*X* and *Y*) measured for each individual are dichotomous, the correlation between the two variables is called the *phi-coefficient.* To compute phi (ϕ), you follow a two-step procedure:

1. Convert each of the dichotomous variables to numerical values by assigning a 0 to one category and a 1 to the other category for each of the variables.

2. Use the regular Pearson formula with the converted scores.

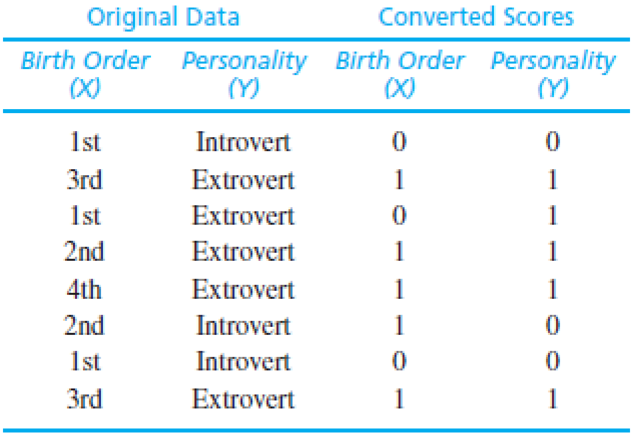
Example

A researcher is interested in examining the relationship between birth-order position and personality for individuals who have at least one sibling. A random sample of *n* = 8 participants is obtained, and each individual is classified in terms of birth-order position as first-born versus later-born. Then, each individual’s personality is classified as either introvert or extrovert. The original measurements are then converted to numerical values by the following assignments:



Birth Order	*Personality*
First-born child = 0	Introvert = 0
Later-born child = 1	Extrovert = 1

The original data and the converted scores are as follows:



Original Data		*Converted Scores*	
Birth Order (X)	*Personality (Y)*	*Birth Order (X)*	*Personality (Y)*
1st	Introvert	0	0
3rd	Extrovert	1	1
1st	Extrovert	0	1
2nd	Extrovert	1	1
4th	Extrovert	1	1
2nd	Introvert	1	0
1st	Introvert	0	0
3rd	Extrovert	1	1

The Pearson correlation formula is then used with the converted data to compute the phi-coefficient.

4.The Point-Biserial Correlation

The point-biserial correlation is used to measure the relationship between two variables in situations in which one variable consists of regular, numerical scores, but the second variable has only two values. A variable with only two values is called a *dichotomous variable* or a *binomial variable*.

To compute the point-biserial correlation, the dichotomous variable is first converted to numerical values by assigning a value of zero (0) to one category and a value of one (1) to the other category. Then, the regular Pearson correlation formula is used with theconverted data.