CHAPTER 7

PROBABILITY AND SAMPLES: THE DISTRIBUTION OF SAMPLE MEANS

SAMPLES AND POPULATIONS

- Summary of the preceding two chapters
  - Whenever *a score* is selected from a *population*, you should able to compute a *z-score* that describes exactly where the score is located in the distribution.
  - *If the population is normal*, you should able to determine probability value for obtaining any individual score.
  - However, z-scores and probabilities that we have considered so far are limited to *situations in which the sample consists of a single score*.

SAMPLES AND POPULATIONS

- In most research studies, much larger samples are used. In these situations, *sample means* rather than a single score is used to answer the questions about the population.
- We transform a sample mean into a z-score. Thus, we find a z-score that describes the entire distribution.
- As always, a *z-score near 0* indicates *a central, representative* sample, *a z-score beyond +2.00 or -2.00* indicates a *extreme sample.*
- Z-scores for sample means can be used to
  - describe how a specific sample is related to all other possible sample
  - look up probabilities for obtaining certain samples

SAMPLES AND POPULATIONS

- Samples provide an incomplete picture of the population
- Although a sample should be representative of the population, some segments of the population cannot be included in the sample.
- Therefore any statistics that are computed for the samples *cannot be identical* to the corresponding parameters of the population.
- *The difference or error between sample statistics and population parameters is called* **sampling error**.
- Two separate samples will be different from each other even if they are selected from the same population.

THE DISTRIBUTION OF SAMPLE MEANS

- **The distribution of sample means** refers to the collection of sample means for *all the possible random* samples of *a particular size (n)* that can be obtained from a population.
- **Notice that** the values in the distribution are not scores, but statistics (sample means)
- A distribution of statistics is called a sampling distribution
- **A sampling distribution** is distribution of *statistics* by selecting *all the possible samples* of a specific size from a population.
- The distribution of sample means is an example of a sampling distribution. It is called **sampling distribution of *M***

THE DISTRIBUTION OF SAMPLE MEANS

- Characteristics of distribution
  - **The sample means** should *pile up around* the population mean. Most of the sample means should be relatively close to population mean
  - It tends to form *a normal-shaped distribution*. In other words, most of sample means should have means close to μ. The frequencies should taper off as the distance between M and μ increases.
  - *The larger the sample size, the closer the sample means should be to population mean*. The sample means obtained with a large sample size should cluster relatively close to the population mean, the means obtained from small samples should be more widely scattered.

THE DISTRIBUTION OF SAMPLE MEANS

- **The Central Limit Theorem**
  - In more realistic situations, it is impossible to actually obtain every possible sample with larger populations and larger samples.
  - *The Central Limit Theorem* makes possible to have a precise description of the distribution of sample means without taking hundreds and thousands of samples.
  - *According to the Central Limit Theorem*, for any population with mean μ and standard deviation σ, the distribution of sample means for sample size (n) will have **a mean of μ** and **a standard deviation of σ/√n** and will approach **a normal distribution as n approaches infinity.**

THE DISTRIBUTION OF SAMPLE MEANS

- The Shape of the Distribution of Sample Means
- The distribution will be perfectly normal *if either of the following two conditions is satisfied.*
  - **The population** from which the sample are selected is a **normal distribution**
  - The number of scores (n) in each sample is relatively large, around 30 or more. **When n>30**, the distribution is almost normal regardless the shape of the original population.
- Mean of the Distribution of Sample Means
  - The mean of the distribution of sample means is *equal to* the mean of the population of scores, μ, and is called **expected value of *M***

THE DISTRIBUTION OF SAMPLE MEANS

- **The Standard Deviation of the Distribution of Sample Means**
- It is called standard error of M. It provides a measure of how much distance is expected *on average* between a sample mean and the population mean
- It serves two purposes
  - The standard error describes the distribution of sample means. It provides a measure of *how much difference is expected from one sample to another.*
  - Standard error measures how well an individual sample mean represents the entire distribution. It provides a measure of *how much distance is reasonable to expect between a sample mean and the overall mean for the distribution of sample means*

THE DISTRIBUTION OF SAMPLE MEANS

- **The Standard Deviation of the Distribution of Sample Means**
- The magnitude of Standard error is determined by two factors:
  - **Sample Size**

    As the *sample size increases*, the error between a sample mean and population mean should *decrease*. Large samples should be more accurate than small samples.
  - **The Population Standard Deviation**
  - *When n=1*, **standard error=standard deviation**
  - Standard deviation is the starting point for the standard error.
  - Standard error= $\sigma/\sqrt{n}$

LOOKING AHEAD INFERENTIAL STATISTICS

- Inferential statistics are methods that use sample data as the basis for drawing general conclusions about population
- The natural difference that exist between samples and populations introduce a degree of uncertainty and error into inferential statistics
- The distribution of sample means and the standard error play a critical role in inferential statistics

STANDARD ERROR AS A MEASURE OF REABILITY

- In most research situations, a single sample is used to make inferences about the population.
- However, if another sample is chosen from the population, the researcher must face a question, "If I had taken different a different sample, would I have obtained different results?"
- This question is about the degree of similarity among all the different sample chose from population
- In this context the standard error can be viewed as a measure of reliability.
- If the standard error is small, a researcher can be confident than any individual sample mean will be a reliable measure
- If the standard error is large, then the researcher can be concerned that a different sample can produce different results