CHAPTER 5

Z-SCORES: LOCATION OF SCORES AND STANDARDIZED DISTRIBUTIONS

INTRODUCTION TO Z-SCORES

- Z-score is a standard score that specifies the precise location of each X value within a distribution.
- The general purpose of z-scores is identify and describe the exact location of every score in a distribution.
- More specifically, z-scores aims to
  - To make raw scores more meaningful. Since the raw scores does not necessarily provide much information about its position within a distribution, they are transformed into z-scores. Therefore, each z-score tells us the exact location of the original X value within the distribution.

  - To standardize an entire distribution. As a result of standardization, different distribution can be equivalent and comparable to each other

Z-SCORES AND LOCATION IN A DISTRIBUTION

- In order to describe the exact location of a score within a distribution, z-score transforms X values into a signed number.
- Z score always consists of two parts:
  - The **sign** tells whether the score is located above (+) or below (-) the mean
  - The **number** tells the distance between the score and the mean in terms of standard deviation.
- Notice that both parts are necessary to describe exact location of a X value.
- The locations identified by z-scores are the same for all distributions, no matter what mean and standard deviation the distributions may have.

Z-SCORE FORMULA

- The numerator of the equation is a deviation score. It measures the distance in points between X and μ and indicates whether the X value is located above or below the mean
- The deviation score is divided by σ because we want the z-score to measure distance in terms of standard deviation units.

$$z = \frac{x - \mu}{\sigma}$$

USING Z-SCORES TO STANDARDIZE A DISTRIBUTION

- The entire distribution can be transformed into a distribution of z-scores that has its own characteristics
  - **Shape**: The shape of the z-scores will be the same as the original distribution of raw scores. Since transforming raw scores into z-scores does not change anyone's position in the distribution, the overall shape will not change.
  - **The Mean**: The mean of the z-score distribution will always have a mean of zero.
  - **The Standard Deviation**: The z-score distribution will always have a standard deviation of 1

USING Z-SCORES TO STANDARDIZE A DISTRIBUTION

- When any distribution (with any mean and standard deviation) is transformed into z-scores, the resulting distribution will always have a mean of 0 and standard deviation of 1
- Because all z-scores have the same mean and the same standard deviation, the z-score distribution is called a standardized distribution.
- Standardized distributions are used to make dissimilar distribution comparable.

OTHER STANDARDIZED DISTRIBUTIONS BASED ON Z-SCORES

- Because Z-scores contain negative and decimals, it is common to transform the scores into a new distribution with predetermined mean and standard deviation that are whole round numbers.
- The aim is to create a new standardized distributions that has simple values for the mean and standard deviation but does not change any individual's location within the location. For instance; IQ tests are standardized.
- The procedure for standardizing a distribution is as follows:
  - The original raw scores are transformed into z-scores
  - The z-scores are then transformed into new X values so that the specific μ and σ are obtained

COMPUTING Z-SCORES FOR SAMPLES

- The same principles can be used to identify individual locations within a sample. For a sample, each X value is transformed into a z-score so that
  - The **sign** tells whether the score is located above (+) or below (-) the sample mean
  - The **number** tells the distance between the score and the sample mean in terms of sample standard deviation.
- When we standardize a sample distribution,
  - **Shape**: The sample of z-scores will be the same as the original distribution of raw scores.
  - **The Mean**: The sample of z-scores will always have a mean of zero.
  - **The Standard Deviation**: The sample of z-scores will always have a standard deviation of 1

INFERENTIAL STATISTICS AND Z-SCORES

- In order to evaluate the effect of the treatment, the treated sample is compared to the original population.
  - If the individuals in the sample are noticeably different from the individuals in the population, it is concluded that treatment has an effect
  - If the individuals in the sample are not noticeably different from the individuals in the population, it is concluded that treatment has no effect
- Z-scores are used in deciding whether a sample is noticeably different from the population.
  - A z-score near 0 indicates that the score is close to the population mean and therefore is representative score.
  - A z-score beyond +2.00 (-2.00) indicates that the score is extreme and is noticeably different from other scores in the distribution.